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Application of optimal power flow to interchange brokerage transaction

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Application of optimal power flow to interchange brokerage transaction

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CHAPTER I. INTRODUCTION

The concept of power system interconnection was introduced in the late 1960's. The reason for interconnection is mainly reliability and security. Through the interconnections, changes in the system load are picked up automatically by other generating sources in the power system [1]. Emergency assistance is also provided when a forced outage of a generating unit occurs. In this way, the impact of disturbances to individual power system is minimized and the spinning reserve requirements are greatly reduced. In addition, the interconnections allow several large systems that peak at different seasons of the year to help each other by interchanging power during their individual peaks. Utilities facing capacity shortages can arrange with other neighboring systems to meet their peak loads. Figure 1.1 illustrates a 3-area interconnected system.

Beyond that, the most important aspect of interconnection is economic gains. Additional savings can be realized through power interchange. The potential for savings exists whenever the marginal generating costs between two areas differ significantly and extra capacity is available. Then, both systems can arrange an economy energy transaction at a specific hour and then share in the total production cost savings. Moreover, the need to build new generating plants and transmission facilities to serve the forecasted load growths are greatly reduced. This saves utilities an enormous amount of money in investment and operation costs.

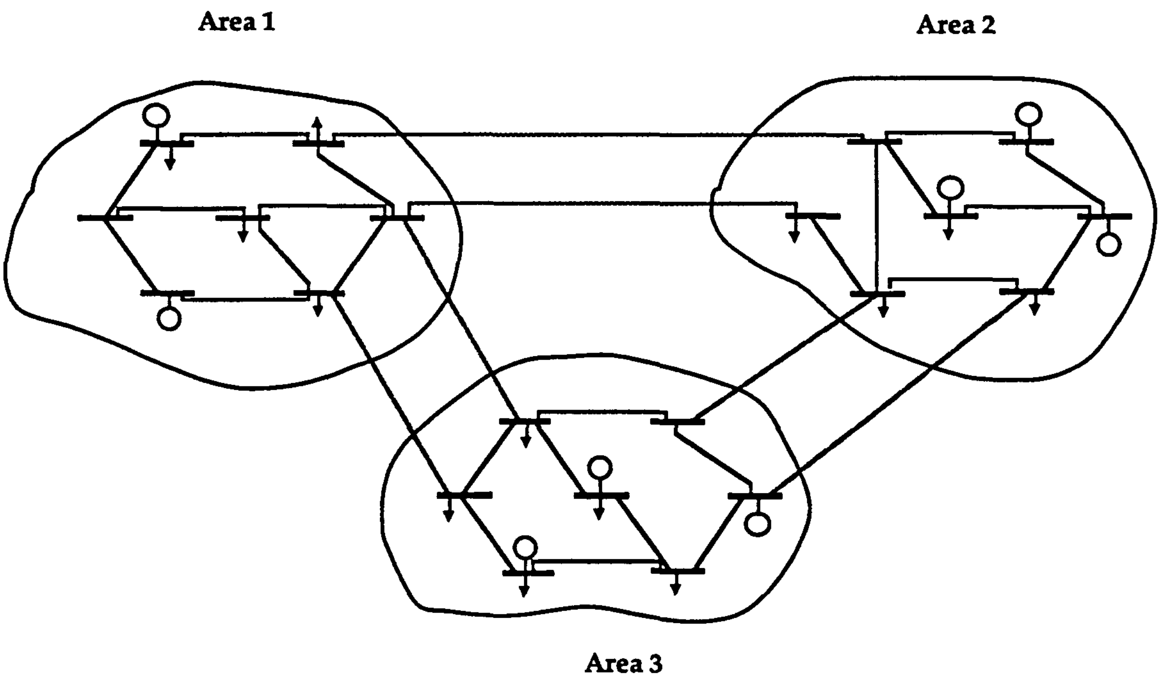

Figure 1.1. Interconnected network of a three-area system

At present, the electric market industry is becoming more competitive than previously. Rising electricity prices, regulatory uncertainties, and financial risks associated with constructing new generating sources and transmission facilities have resulted in efforts to reduce the short-term energy costs. One practical way is through interchange. While bilateral transactions allow two adjacent utilities to do interchange efficiently and satisfactorily from an economic point of view, it becomes impractical when trading involves several utilities simultaneously. Complexity frequently arises as how to maximize the energy transfer and to allocate savings satisfactorily to all utilities involved.

To reduce this confusion, an efficient procedure is needed that will maximize the energy transfer level using an on-line implementation. One method is called interchange brokering; it is designed to provide for hourly non-firm economy interchange utilizing on-line generating units only. A central broker acts as the coordinator to arrange purchases and sales such that all participants save and profit equally from the transaction.

Brokerage business works well especially in a full competitive market. In almost all commodities exchanges, brokers exist to make a market. One example is the agriculture commodity exchange. The farmers in the Midwest do not concern themselves with price negotiations with foreign countries, shipping and handling, or export-import regulations. Similarly, the prospective consumers do not go around the country to locate the low-priced resources. Instead, both the producers and the consumers contact a broker who acts as a middleman in the marketplace. This broker solves transportation problems, handles regulatory concerns, accepts inventory risks, and establishes market information for both the producers and the consumers. Therefore, the broker provides services that are worthwhile to the participants in the commodity market.

The trend toward a deregulated and competitive market is a topic gaining much attention in the U.S. electric market industry presently. Regulators see direct competition as necessary to achieve economy efficiency. Through the passage of the public utility regulatory policies act (PURPA) in 1978, regulators allow non-utility generators (NUG's) and independent power producers (IPP) to enter the competition. This, however, should be followed by increased access to utility transmission systems and deregulated pricing of transmission services. One possible way that was recently proposed by the Federal Energy Regulatory Commission (FERC) is through the open transmission access. The FERC brought up this proposal because they feel that the interconnected networks have not been utilized efficiently. With the trend toward the deregulated and more competitive market, it appears that the wholesale power market may gradually shift to interchange brokerage operation in the future.

This thesis provides a ground work for evaluating interchange brokerage transactions. The necessary indices for evaluating the interchange are obtained from an optimal power flow (OPF) solution. The OPF is solved using an augmented Lagrangian (AL) technique - a nonlinear programming approach that combines dual and penalty methods. A steady-state condition is assumed during the hourly brokerage interchange transactions. No generator or line outage is considered. Transmission losses because of inadvertent power flows are also not included.

Chapter II reviews the OPF literature and the energy brokerage practice in the United States. It also discusses the marginal cost theory that is essential for evaluating economic power transactions. The theory of the non-linear programming (NLP) in OPF is presented in Chapter III. Chapter IV presents the research results. Finally, Chapter V gives conclusions and recommendations for future work.

CHAPTER II. REVIEW OF LITERATURE

Today's power interchange activities are marked by a significant increase in the number of sales and purchases. Utilities that see interchange as one effective way to reduce short-term energy costs have maximized their usage of the interconnected system. This has increased transmission line loadings. Transmission lines are often forced to operate at their maximum limits. To ensure security and reliability, power system performance must be evaluated continually and the power system bottle necks must be eliminated or reduced. To assist the power system operator in meeting these objectives, an optimization technique that gives a complete analysis of the system is sought; this technique must identify generation availability, internal transmission constraints, and bus voltage constraints.

Previously, two different optimization approaches were considered for solving power interchange evaluation, namely optimization using the distribution factor (DF's) and the optimal power flow (OPF).

2.1. Review of DF's and OPF Approaches

Landgren, Terhune, and Angel [2] developed a method to determine the transmission network interchange capability. The method used a so-called power transfer distribution factor (PTDF) and a load outage distribution factor

(LODF) to analyze the system under normal and emergency operating conditions. PTDF was used to identify the limiting circuit elements that were most likely become overloaded. The LODF helped to identify key line, tower, or generator outages that were most likely to cause other circuit elements to become overloaded.

G.L. Landgren and S.W Anderson in [3] presented a linear programming model to solve the simultaneous interchange capability problem. The model implemented PTDF and LODF just mentioned to calculate the sensitivity factors that allow the operator to determine a safe operating margin.

Not many papers with PTDF applications have been published recently [4]. The main disadvantage is that DF approach does not address both real and reactive (MVA) line flow or bus voltages.

2.1.1. Evolution of the OPF

The power industry today requires the development of more complex nonlinear power system models and optimization techniques for solving them, rather than the simple approach described above. These techniques are called the OPF. The OPF has been successfully applied to various power system problems for many years; these include minimizing system MW losses or system generation cost, redispatching generation, and performing contingency analysis.

Typical constraints modeled in the OPF are:

- MW, MVAR or MVA flow on transmission lines and transformers

- Buses voltage magnitudes and angles
- Generator MW and KV
- Regulating transformer adjustments

Equation (2.1) defines the general structure of a typical OPF model [4]. Common OPF objectives are: minimize the operating costs, minimize the active power transmission losses, minimize the number of controls rescheduled, and minimize the deviation from a specified point [5].

$$\begin{aligned}
&\text{Minimize} && f(\Delta P_g) \\
&\text{subject to} && h(\Delta P_g) = 0 \\
& && \Delta g^{min} \leq \Delta g(\Delta P_g) \leq \Delta g^{max} \\
& && \Delta P_g^{min} \leq \Delta P_g \leq \Delta P_g^{max}
\end{aligned} \tag{2.1}$$

where ΔP_g - control variables

$h(\Delta P_g)$ - a set of equality constraints

$g(\Delta P_g)$ - a set of inequality constraints

The control parameters in power system operation may represent any of the following: real and reactive power generation outputs, generator voltages, regulating transformers, or shunt reactors and capacitors. Equality constraints generally represent the active and reactive bus power injection equations, which must be satisfied in all power flow solutions. Inequality constraints represent the upper and lower limits on the power system quantities. This includes all the 'soft' and 'hard' operating limits. The 'soft' limit implies that any violation near the constraint may still be tolerated. They generally refer to the operating constraints such as branch current and MVA flows, spinning MW and MVAR reserves, area MW interchanges, and bus voltage angle separations. In contrast, 'hard' limits suggest a strict operating region. These constraints must be satisfied because any violations to these limits may result in severe damages to the devices. The 'hard' limits represent the ranges of physical devices such as unit's active and reactive power generating capabilities.

2.1.2. State-of-the-Art OPF Techniques

The present state-of-the-art in OPF techniques involve either linear programming (LP) or the Newton algorithm [4] LP has received significant attention because of its reliability, efficiency, and solution speed [6]. Addition or removal of constraints is easily incorporated into the LP model to obtain new solutions. Through sensitivity analysis, new operating points can be evaluated without having to reformulate and resolve a complete problem. In addition, when only binding constraints are enforced and relaxed constraints are removed as in the dual LP method, the computer memory requirements are significantly reduced. LP also detects and solves infeasibility quickly. A major limitation with LP is the linearization of the objective function and constraints. A tradeoff exists between the solution accuracy and computation speed.

Originally, the Newton method was used only for power system planning applications because of its moderate convergence speed [7]. Recently, the decoupling of the method has achieved the acceptable accuracy and computational performance for on-line implementation. One difficulty with Newton approach is its inability to detect and handle infeasibility. This method also requires the computation of the second partial derivatives of the power flow equations and other constraints (the Hessian matrix). Sparsity techniques must be exploited to make the practical use of this method. Moreover, its convergence performance is sensitive to tuning.

2.1.3. Motivation behind this Research Work

One important aspect of the choice of the OPF is its ability to overcome infeasibility when the operating limits cannot be tolerated. Because of the complexity of the nonlinear power system equations that are involved, a local optimum may not exist. Instead of stopping unsolved, the OPF algorithm should provide the best possible solution to the decision maker [5]. This is a requisite for an energy management system (EMS) implementation. Inoperable constraints should be recognized and possibly be relieved so that a new feasible operation region may be determined. If no feasible region is found, the algorithm should still provide the closest possible solution. This allows the operator to make any necessary decisions to relieve these constraints and determine a new feasible operating region. Unfortunately, most OPF methods such as Newton and LP techniques are deficient in this aspect. They are not able to provide any solution to an infeasible problem.

Another important aspect is that the OPF solutions must be insensitive to the initial starting point. Normally, when different initial points are used, different optimal solutions are obtained. All numerical methods that use an iterative procedure exhibit this characteristic. The method, however, should reach consistent solutions despite the starting points In other words, the differences in the solution must be insignificant and the solution must agree with the true global minima.

In addition, if multiple local minimas exist in a feasible region, more computational efforts may be required. A heuristic type of search must be performed to find the local minimas and select the best one.

Lastly, the choice of the OPF should be expanded to include future long-term interchange transactions. The long-term transaction analysis requires a Unit Commitment (UC) evaluation. Since UC constraints are complex and nonlinear by nature, the nonlinear constraints may have to be replaced with many piecewise linearized segments if an LP algorithm is used. The solution procedure then becomes very time consuming. Therefore, the LP algorithm may be impractical.

The AL method fits nicely with the above requirements. The non-linear constrained problem can be modeled easily with this method, since the AL technique is naturally a non-linear programming technique. Moreover, the AL method handles infeasibility rapidly. It also gives a consistent solution to a given problem. The mathematical details of AL approach will be discussed in Chapter 3.

2.2. Operating Problems with Parallel Flow

Parallel flow is a natural phenomenon in interconnected system operations [8]. It exists because power flow follows the physical laws and not the contract paths; that is, the electricity flows over the lowest impedance paths despite the ownership of the transmission facilities.

Parallel flows often increase transmission line loadings. As a result, owners are prevented from using their own transmission systems for individual transfers and purchases. In addition, utilities experiencing such increased flows also incur additional system losses. Therefore, these utilities, not only suffer from the reliability, but also the economic disadvantages

Greater distances between the source and the point of utilization generally produce more widespread parallel flow effects. These effects often limit the power transaction schedules among the local systems along the path. They can be minimized by installing series capacitors or phase-shifting transformers. In more severe cases, the AC networks are replaced by the High Voltage Direct Current (HVDC) transmission lines. One example is the Pacific DC line that was built between the Northwest and Southern California to preserve power transfer capabilities between the two systems [8].

Phase-shifting transformers have also been used extensively to maintain scheduling capabilities on the interface tie-lines. One advantage is that the transformer tap changes can be calculated in advance, depending on the owner's objectives [9]. The regulating transformer variables are incorporated in the OPF solution by a set of equations that describe the controller adjustments (section 3.5).

2.3. Spot Pricing of Electricity

Traditionally, commercial and industrial electric rates have been based on the average incremental cost of generation, transmission, and distribution They generally do not vary from season to season, day to day, or hour to hour, although there are differences in the instantaneous costs. Under this pricing regime, the rates are calculated as follows. First, all the utility's costs are summed; these costs usually include capital, depreciation, taxes, operating and maintenance (O&M), fuel, and overheads. Then the costs are averaged over all the customers total yearly, energy use to yield the average price of a kilowatt-hour (KWh).

The average pricing, however, does not give any economic incentives to customers to adjust their electricity consumption. This happens because customers see the same prices whether it is a high-cost period or a low-cost period. Therefore, they are unwilling to reduce their demands during peak hours. As a result, the utility must invest billions of dollars in equipment that generally sits underutilized most of the time to ensure reliability [10].

To use the generation and network capacity efficiently, utilities have sought to increase customer-utility cooperation in the electric market industry. Customers will be given more flexibility to manage their own demand and budgets. They may choose whether to reduce the demand during peak hours or pay high spot prices to get energy. Similarly, those with lower priority usage may choose to shift their energy use from high spot price periods to low spot price periods. This pricing scheme also brings relief to the utility during the peak loads. Thus, both the customer and the utility would benefit from spot pricing. Nevertheless, if customers do not respond to spot prices, the utility sees the same demand pattern as before and is no worse off.

2.3.1. Behavior of Hourly Spot Prices

In the energy marketplace, all utility-customer transactions must be coordinated and operated using a self-consistent value. This gives a notion of the hourly spot price [10]. The hourly spot price is determined by the supply and demand conditions, generation availability and costs, and transmission and/or distribution network availability and losses at that hour.

The hourly spot price is defined in terms of marginal (or incremental) cost that encompasses the total cost of providing electricity to all customers and is evaluated with the following constraints:

- the total generation equals the total demand plus losses.

- the total demand during one hour cannot exceed the capacity of all the power plants available at that hour.
- energy flows and losses on a network are specified by the physical laws
- energy flows over a particular line cannot exceed specified limits without causing operating system problems.

The hourly spot price components are the sum of the following: the marginal generation cost, the marginal generation maintenance cost, the generation quality of supply costs, the marginal network losses, the marginal network quality of supply costs, and the revenue reconciliation.

Marginal generation fuel and maintenance costs consist of two components. The first one is the derivative of generation fuel and maintenance costs with respect to demand during one hour. The second is the net purchases the utility makes with interconnected utilities. Normally marginal generation and maintenance costs increase with total demand. This also depends on plant outages, water (hydro) availability, and external purchase-sale opportunities. The marginal network cost component arises from the electricity losses on transmission and distribution, depending on the customer location. Generation quality of the supply cost component is very small or zero most of the time and increases rapidly as generating capacity limits are being approached. By analogy, network quality of the supply cost component becomes large when the network capacity is stressed. To ensure that utilities do not make or lose too much money, revenue reconciliation components should be considered

2 3.2. Revenue Reconciliation

Short-run marginal costs or spot prices often produce too little revenue to recover the investments made by the utility [10]. Moreover, it does not give enough incentives to the utilities to build additional generation or transmission capacity to serve loads. For example, additional reactive power compensation equipment must be purchased to serve increased reactive power demands and to maintain the desired voltage levels. This equipment may cost the utility millions of dollars However, the added costs may not even be repaid by customers under the spot pricing scheme because there are no charges for generating the extra marginal reactive power. As a result, the return of investments is often not recovered.

Therefore, revenue reconciliation is necessary to ensure that electric utilities do not lose (too much) money. Revenue reconciliation may include interest, debt payment and rate of return of the existing power plants and the network. Other costs that must be compensated include fuel and generation maintenance costs, network maintenance costs, administration, metering, billing, and other fixed costs. Reference [10] discusses various approaches to calculate revenue reconciliation.

2.4. Review of Energy Brokerage System

In the past, transactions occurred only between adjacent utilities. They were of the non-firm hourly type of transaction. The potential sellers offered energy when extra capacity was available. They did not guarantee any capacity but sold energy when it was convenient for them. This transaction type is also called economic energy interchange. The purpose is to improve the short-term production efficiencies by minimizing the total fuel costs of both parties Therefore, whenever a company has a production cost significantly lower than its neighbors, the transaction can take place. Then, the company with the lower incremental cost raises its power generation by the agreed amount. The scheduled amount of energy is exported to the company with the higher incremental cost. Simultaneously, the higher cost company lowers its own native generation level. The energy transfer then proceeds until the end of the agreed hour has been reached. Then both companies adjust their own generation back to their normal operating levels.

Inter-utility power interchange practice, however, has changed significantly over the last ten years. Transaction levels have increased not only in magnitude and frequency, but also in complexity; this means that a utility may have several transactions take place simultaneously under complicated circumstances.

To ease the complexity and to achieve maximum economical benefits, several utilities have formed power pools or engaged in energy brokerage transactions. A power pool is a group of utilities that operates at a single incremental cost λ. Power pooling allows a utility to maximize its economic operation better than if operating individually. Moreover, it also helps member companies in providing back-up during emergencies, sharing spinning reserves, coordinating unit commitment and maintenance scheduling [1] Nevertheless, the pool's agreements are usually complex. This happens because each member attempts to obtain greater benefits from the pool operation. The more members try to push for maximum economic operation the more complicated the agreements become. Also, complexity may arise in dealing with regulatory agencies if the pool operates in more than one state. Energy brokering provides an alternative

Energy brokering allows each utility to retain control of its own generating facilities and makes its own unit commitment decisions. It does not impose any interchange contracts; the final decision on whether to enter a transaction is left to the individual companies [1]. Like the power pool, it may serve to coordinate planning, construction, and utilization of generation and transmission facilities of the system. This, however, depends on the extent of cooperation defined in the agreements.

Broker operation is based on two aspects. First, it ensures that no energy remains untraded when there is a potential saving in the transaction. In other words, as far as the seller's incremental cost is lower than the buyer's decremental cost ($\lambda_s < \lambda_b$), a transaction will take place. Second, the total savings are distributed among participating utilities according to a certain policy. The saving calculation is simple because the distribution of the total savings is determined only by the cost of the energy sold and the avoided cost of purchasers. This is different from a pool's operation where the saving distribution policies also depend on the percent of peak, the percent of reserve, and the number of customers served by each company

Energy brokering resembles a decentralized spot market for electricity since it introduces greater competitions in the wholesale power market. The transaction price is not only based on the utility's bid alone but also by the aggregate supply and demand conditions. That is, the price of energy is determined by the energy price offered by the seller and the utility's ranking among all bidders.

Figure 2.1 illustrates a simple brokerage operation. A central broker acts as the transaction coordinator. At 15 minutes before the hour, each participating

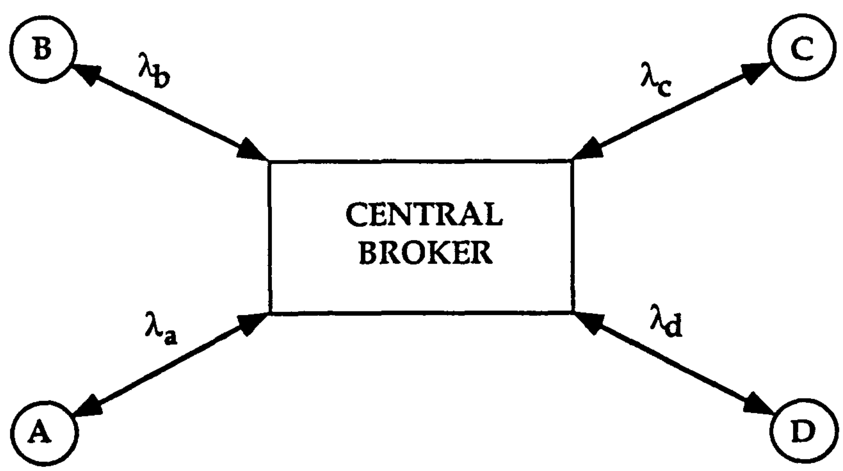

Figure 2.1. Energy brokerage operation of a four-company system.

utility (A, B, C, D) submits its energy requirement and the price quotation to the central broker. Depending on the interchange agreements, quote components may cover either incremental fuel cost, incremental O & M (if available), and/or incremental transmission loss adjustment costs. The broker evaluates the purchase and sale bids according to certain rules and determines the possible transactions that will achieve maximum gains for all participants. He then communicates the results and makes recommendations to each participant. Finally, each participant decides whether to accept the proposed transaction. Any participant that sees an unforeseen need coming may immediately withdraw from the transaction.

2.4.1. Determination of the Optimal Economy Energy Schedule

Two methods are commonly used to determine the optimal economy energy schedule:

[1] **Pool-average matching**: match each decremental cost (buy quote) with the average incremental cost, and each incremental cost (sell quote) with the average decremental cost.

[2] **High-low matching**. match the highest decremental cost with the lowest incremental cost.

Pool-average matching requires multilateral contracts where all participants can trade with one another. High-low matching requires only bilateral contracts that encourage more eligible companies to participate in the transaction. The contract conditions are usually agreed by both parties These conditions may include emergency assistance, make-up losses, wheeling services, and wheeling cost allocations.

The high-low matching procedure begins by matching the lowest sell offer with the highest buy bid. Then, the next lower sell offer is matched with the next higher buy bid. This process continues until all offers are covered. The matches are arranged such that the first match captures the largest net savings. The second match maximizes the next largest savings, and so on until all savings are exhausted. A minimum spread between matched buy and sell quotes may be established to compensate for any cost underestimations in all quotes; this ensures that buyers and sellers in the broker scheme realize profits from all transactions.

Both the high-low method and the pool-average method yield the same total amount of savings, but the distribution of the savings differ.

2.4.2. Distribution and Fair Allocation of Savings

Dividing savings equitably is essential, but there is no one best method. Fair allocation is negotiated or agreed upon among participants, usually on the contractual agreement basis. Various methods have been reported to allocate savings [11]. The split-the-savings method is most frequently used in economic power interchange today. It ensures that all involved participants in the transactions profit equally. The transaction price is set at the seller's cost of generation plus one-half of the purchaser's savings in operating costs.

Equations (2.2) and (2.3) describe both the saving and the transaction price respectively [1].

$$\text{Savings} = (\lambda_p - \lambda_s)\, I_{ij} \qquad (2\ 2)$$

$$TP = \lambda_s + (\lambda_p - \lambda_s) / 2 \qquad (2.3)$$

where

λ_s - Seller's incremental cost

λ_b - Buyer's decremental cost

I_{ij} - Amount of interchange between area i and j

The transaction price TP in equation (2.3) is simplified to

$$TP = (\lambda_p + \lambda_s) / 2 \qquad (2.4)$$

2.4.3. Interchange Brokerage Procedure

The interchange brokerage procedure may be summarized as follows:

A. Collect all the buy and sell quotes from each participant 15 minutes before each hour.

B. Rank quotes.

- Sell: LOW to HIGH
- Buy: HIGH to LOW

C. Set up transaction based on:

- Highest buy to lowest sell
- Existing contracts

  If companies A and B do not have a bilateral contract, then they cannot arrange an economy interchange with each other.

- Transmission constraints

  Previously, any transaction that violates the transmission constraints is automatically omitted and the next match is determined. The optimization procedure can be improved to include the tie-line constraints so that the transaction with the line constraint violation is still eligible; the total amount of energy, however, is now reduced to the maximum capacity of the interconnections.

- Minimum allowable quotation spread

  There are three options in the high-low matching algorithm:

  [1] Match buyers and sellers until there is no spread between the incremental quotes ($ Sell/MWh) and the decremental quotes ($ Buy/MWh)

  [2] Match until a minimum spread is reached.

  a. Minimum spread set at a $/MWh value.

  b. Minimum spread set at some percentage of the transaction price i.e. ($ Buy/MWh - $ Sell/MWh)

  [3] Consider spread between quotes and transaction price.

  a. $ Buy - $ TP

  b. $ TP - $ Sell

D. The savings and the transaction price are calculated as described by equation (2.2) and (2.4) respectively.

$$\text{savings} = (C_p - C_s)\, I_{ij}$$

$$TP = (C_p + C_s) / 2$$

2.4.4. Brokerage Refinements

Effects of wheeling and losses must be considered when the transaction involved a third party. Wheeling costs are fixed charges paid to an intermediate utility for its transmission service; they are intended to cover capital investment costs and are usually charged at $/MWh. Loss costs are variable charges based on an intermediate utility's incurred losses. The total losses may increase or decrease as a result of additional MW flow through the third party's transmission system. These losses may be reduced if the transaction is in the opposite direction to the prevailing flows. Then, the third party that benefits from this transaction should pay the other parties since the transaction reduces the total system losses. The loss charges usually are charged as some percentage of MW interchange and priced at transaction price or third-party incremental cost. Both wheeling and loss charges are costs incurred by and must be compensated for the third party. These costs may be either divided between the buyer and the seller, or paid entirely by the buyer upon agreement.

CHAPTER III. PROPOSED SOLUTION METHODOLOGY

3.1. Mathematical Background

The augmented Lagrangian method (AL) is a nonlinear programming technique that combines the dual and the penalty function methods [12]. The AL method serves as a mean to eliminate the disadvantages that these two previous methods have inherently. According to the dual theorem, when the Lagrangian multipliers associated with the constraints are optimized, the optimal solutions of the original problem are also found. These multipliers are known as the shadow prices and often have meaningful interpretations such as prices associated with constraint resources. The dual method, however, requires that the functional constrained problem exhibits a locally convex structure in order for the dual function to be defined. Otherwise, a duality gap may occur and this method will fail to find the optimal solutions. Moreover, the iterative procedure converges only moderately fast Although the best unconstrained minimization search technique is employed, many unconstrained minimization solutions must be evaluated. As a result, the dual approach has lacked applications in many classes of problems.

The penalty function method solves the non-linear constrained problem by performing a sequence of unconstrained minimizations. The characteristic of this method is that the objective function is augmented with a penalty factor that prescribes a high cost for violating any of the constraints. This penalty factor determines the severity of the penalty and consequently the degree to which the unconstrained problem approximates the original constrained problem. Unlike the dual method, the penalty method handles nonconvex problems very well. Nevertheless, the penalty approach is sensitive to round-off errors and it becomes increasingly ill-conditioned as the penalty factors approach infinity

3.2. Augmented Lagrangians

To overcome the difficulties that the penalty method has, a multiplier term is introduced as with the dual method; this allows the penalty method to approach the optimal solutions for substantially lower values of the penalty factors and thus avoids the ill-conditioning problem. Moreover, at sufficiently large penalty factors, the Lagrangian function becomes locally convex near the solution points [12] This approach, which combines penalty function method and dual method, is called the augmented Lagrangian method.

3.2.1. Optimality conditions

The augmented Lagrangian method must satisfy a set of optimality conditions when solving a non-linear constrained problem. These optimality conditions are also known as the Karush-Kuhn-Tucker (KKT) conditions and are introduced next.

Consider the optimization problem

$$\min_{x} \; f(x) \tag{3.1}$$

such that

$$g_i(x) \leq 0, \qquad i=1, 2, ..., r \tag{3.2}$$

where

$$x = [\, x_1, x_2, ..., x_n]^T$$

Assume that the Lagrangian multipliers λ_i exist. The Lagrangian function for the above problem becomes

$$L(x,\lambda) = f(x) + \sum \lambda_i \, g_i(x) \tag{3.3}$$

Then at the point x* satisfying

$$\min_{x} \; f(x) = f(x^*) \tag{3.4}$$

such that

$$g_i(x^*) \leq 0, \qquad i = 1, 2, ..., r \tag{3.5}$$

The following conditions must hold [13]

$$\nabla_x L(x^*,\lambda^*) = 0$$
$$\nabla_\lambda L(x^*,\lambda^*) \le 0$$
$$(\lambda^*)^T g(x^*) = 0$$
$$\lambda^* \ge 0 \tag{3.6}$$

In addition, at the optimal points (x^*, λ^*), the Lagrangian function $L(x^*, \lambda^*)$ must exhibit a saddle point [13] whose condition is characterized by

$$L(x^*,\lambda) \le L(x^*,\lambda^*) \le L(x,\lambda^*) \tag{3.7}$$

Equation (3 7) means that if (x^*, λ^*) exhibits a saddle point for the Lagrangian function, then x^* is the solution for the constrained primal problem.

3.2.2. Duality and the Saddle-Point Condition

Define the dual function as

$$h(\lambda) = \min_{x} L(x,\lambda) \tag{3.8}$$

Assume that there is a set of λ such that $L(x,\lambda)$ has a finite minimum with respect to x. Then for all x satisfying $g(x) \leq 0$, the dual function always provides a lower bound to the function $f(x)$ such that $h(\lambda) \leq f(x)$. In addition, since the dual function provides a lower bound to $f(x)$, it follows that the largest, lower bound must occur at the maximum value of $h(\lambda)$.

According to the duality theorem [13], the point (x^*, λ^*) is a saddle point of the Lagrangian function defined previously if and only if

(i) x^* solves the primal problem

(ii) λ^* solves the dual problem

(iii) $f(x^*) = h(\lambda^*)$

where

Dual function:

$$h(\lambda) = \min_{x} L(x,\lambda)$$

$$D = \{\lambda \mid h(\lambda) \text{ exists, and } \lambda \geq 0\} \tag{3.9}$$

Dual problem·

$$\max_{\lambda \varepsilon D} h(\lambda) \tag{3.10}$$

3.2.3. Augmented Lagrangian Formulation

Let the standard minimization constrained problem be

Minimize $F(x)$

Subject to

$$P_i(x) - a_i = 0 \qquad i = 1, 2, .., noe$$

$$Q_j(x) - b_j \le 0 \qquad j = 1, 2, ..., noi$$
$$c_k - x_k \le 0 \qquad k = 1, 2, ..., nolb$$
$$x_k - d_k \le 0 \qquad k = 1, 2, ..., noub \qquad (3.11)$$

The AL formulation for the above constrained problem can be written as

$$\begin{aligned} \textbf{AgLag} = {} & F(x) + \lambda_1 (P_1(x) - a_1) + \mu_1 (Q_j(x) - b_j) \\ & + x_{lb} (c_k - x_k) + x_{up} (x_k - d_k) \\ & + w_1 (P_1(x) - a_1)^2 \\ & + (w_2 \text{ or } w_3) [(Q_j(x) - b_j)^2 + (c_k - x_k)^2 + (x_k - d_k)^2] \qquad (3.12) \end{aligned}$$

where w_2 and w_3 represent the penalty factors for the active inequality constraint and the violated inequality constraint respectively [12].
The gradient of AgLag with respect to x is as follows

$$\begin{aligned} \nabla\textbf{AgLag} = {} & \nabla F(x) + [\lambda_1 + 2w_1(P_1 - a_1)] \nabla P_1(x) + [\mu_j + 2w_2(Q_j - b_j)] \nabla Q_j(x) \\ & - [x_{lb} + 2(w_2 \text{ or } w_3)(c_k - x_k)] \\ & + [x_{up} + 2(w_2 \text{ or } w_3)(x_k - d_k)] \qquad (3.13) \end{aligned}$$

The above AL function is treated as an unconstrained minimization problem and is solved iteratively. At each iteration, the multipliers and the penalty factors are updated to improve the convergence. When the penalty factors are sufficiently large, the Lagrangian function becomes locally convex; at this point, it can be guaranteed that the indefinite Hessian matrix of the augmented Lagrangian function becomes positive definite. As (λ, μ, x_{lb}, x_{up})

approach (λ^*, μ^*, x_{lb}^*, x_{up}^*), the solutions finally converge to [x_1^*, x_2^*, ..., x_n^*] The AgLag(x^*, λ^*, μ^*, x_{lb}^*, x_{up}^*, w) reaches the unconstrained local minimum of AgLag(x, λ^*, μ^*, x_{lb}^*, x_{up}^*, w) [12]

3.2.4. Multiplier Updating Rules

Recall that the dual function provides a lower bound to the original function f(x). Moreover, at the largest Lagrangian multipliers (λ^*, μ^*, x_{lb}^*, x_{up}^*), the optimal solutions of the primal problem are found. Unfortunately, these multipliers are not known in advance. Therefore, the Lagrangian multipliers must be updated according to certain procedures to reach the optimal Lagrangian multipliers.

Consider the unconstrained AgLag at the first iteration. The unconstrained problem is solved iteratively until the gradient of the AL function becomes zero.

$$\nabla \mathbf{AgLag}^1 \cong 0 \tag{3.14}$$

The KKT necessary conditions, however, require that the gradient of the Lagrangian function to be zero. That is,

$$\nabla La = \nabla F(x) + \lambda_i \nabla P_i(x) + \mu_j \nabla Q_j(x) - x_{lb} + x_{up} = 0 \tag{3.15}$$

Therefore, to satisfy the optimality condition, the multipliers (λ^1, μ^1, $x_{lb}{}^1$, $x_{up}{}^1$) must be updated such that

$$\nabla La^1_{new} - \nabla AgLag^1_{old} = 0 \qquad (3\ 16)$$

Based on this concept, the multiplier updating rules yield equations (3.17) - (3.20).

For the equality constraints,

$$\lambda_i^{new} = \lambda_i^{old} + 2w_1(P_i\text{-}a_i) \qquad i = 1,2,\ldots,noe \qquad (3.17)$$

For the inequality constraints,

$$\mu_j^{new} = \mu_j^{old} + 2\ (w_2 \text{ or } w_3)\ (Q_j\text{-}b_j) \qquad j = 1,2,\ldots,noi \qquad (3.18)$$

For the lower bound constraints,

$$x_{lb} = [x_{lb} + 2\ (w_2 \text{ or } w_3)\ (c_k\text{-}x_k)] \qquad k = 1,2,\ldots,nolb \qquad (3.19)$$

For the upper bound constraints,

$$x_{up} = [x_{up} + 2\ (w_2 \text{ or } w_3)\ (x_k\text{-}d_k)] \qquad k = 1,2,\ldots,noub \qquad (3.20)$$

3.2.5. Penalty Updating Rules

The penalty factors are designed to make the indefinite Hessian matrix become positive definite. At each iteration, they are updated to improve the convergence of the algorithm [14]. At sufficiently large penalty factors, the existence of the AL saddle point is guaranteed. The constant penalty terms w_1, w_2, and w_3 at each iteration are increased by a factor ß ≥ 1 until the given upper bounds w_{imax}'s are reached. In summary, the penalty weight update rule is given as

$$w_k^{new} = \begin{cases} w_{imax}, & \text{if } \beta w_k \geq w_{imax} \qquad i = 1,2,3 \\ \beta w_k, & \text{otherwise} \end{cases} \tag{3.21}$$

It is important that the initial values of w must be sufficiently large to avoid an unbounded and an infeasible solutions. If the initial values of w are too small, then the objective function f(x) decreases at a rate faster than that of the penalty terms $w_i p_i$'s can increase; the subproblem becomes unbounded and defective. In contrast, if w's are initially too large, then ill-conditioning may result.

The choice of the initial penalty values, however, depends on the case study and is also determined by experiences. Often, the following equation may be used to approximate the initial penalty terms.

$$w_k = \frac{2\,F(x_o)}{\sum_{i}^{noe} eqc_i^2 + \sum_{j}^{noi} ineqc_j^2 + \sum_{m}^{nolb} xlbd_m^2 + \sum_{n}^{noub} xubd_n^2} \tag{3.22}$$

If w^k turns out to be less than one, then w^k must be fixed to a value that is greater than one.

3.2.6. General Structure of the AL Optimization Technique

Figure 3.1 shows the general structure of the AL algorithm. First, the values of the parameters are initialized. The penalty factors are calculated and the augmented Lagrangian function is formed. The unconstrained minimization subproblem proceeds as follows. Based on the initial conditions, the search direction r^k is determined. Next, the algorithm finds the point $x^k + \rho\, r^k$ such that ρ minimizes the AgLag function along the direction of the search from x^k. Following the AL minimization subproblem, a set of stopping conditions is tested. If all optimality conditions are satisfied, then the search stops. Otherwise, the multipliers and the penalty factors are updated, and the same procedures starting from the new search point x^{k+1} are repeated.

Recently, the author wrote a Fortran program to the test this algorithm. The program used the Nelder and Mead simplex method to perform the

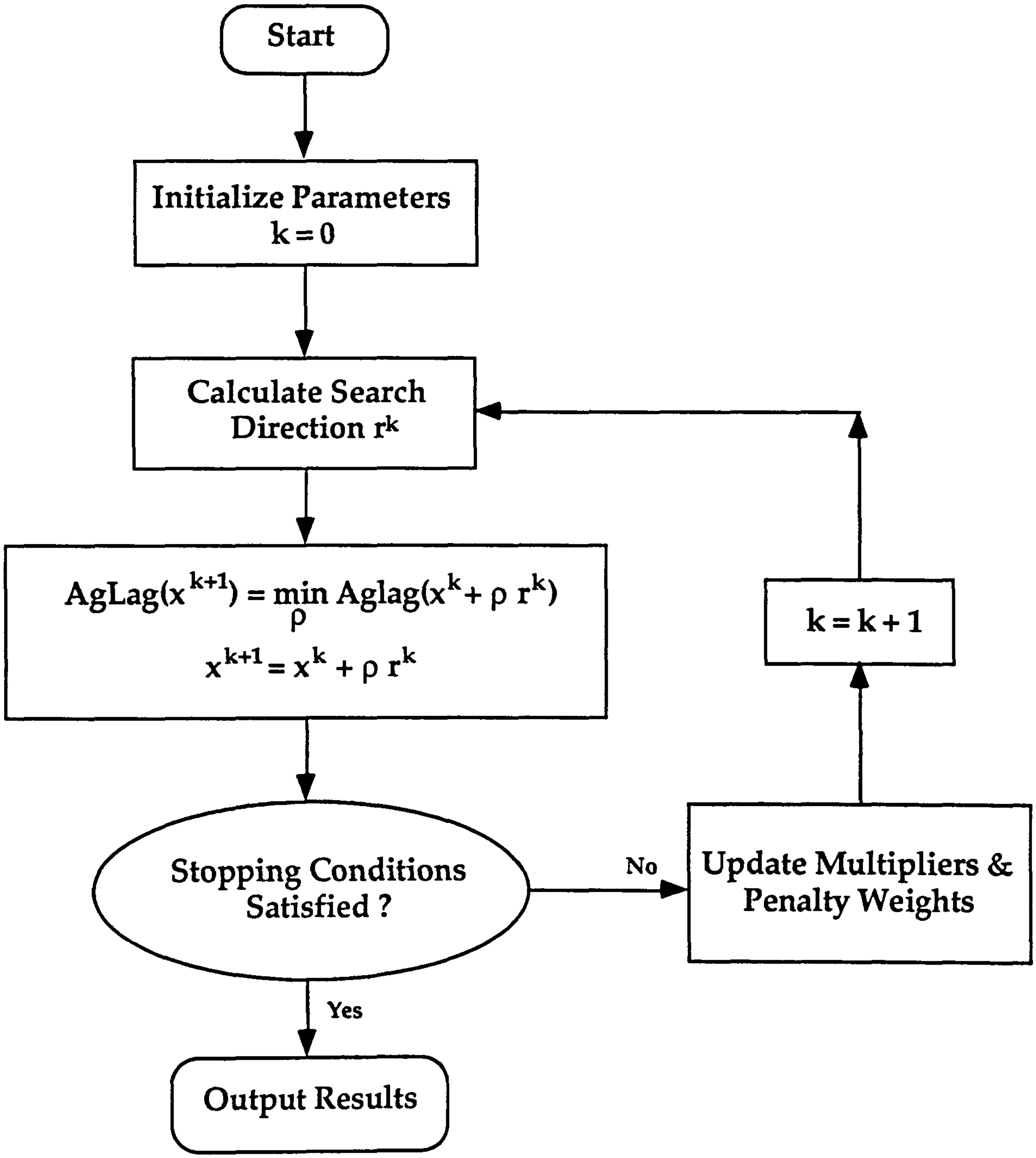

Figure 3.1. Non-linear optimization via augmented Lagrangians

unconstrained minimizations At each iteration, a collection (simplex) of (N+1) design vectors are updated and the worst one is removed.

One highlight of the Nelder and Mead simplex method is that it does not require the derivatives of the function. This greatly simplifies the testing since the derivatives of the OPF problem are difficult to formulate.

This research seeks only to provide the theoretical foundation of the AL OPF in solving power system operating problems. Ease of implementation, rather than high speed computation, is sought in testing the algorithm. The simplex method fits this purpose and was therefore implemented. Consequently, the simplex method is slow. To make the AL OPF efficient and applicable, a higher performance algorithm should be selected for solving the unconstrained subproblem.

3.3. Formulation of OPF

The OPF formulation is presented in equation (3 23). The objective function minimizes the total cost of generation. The equality constraints represent the power injection at each bus. The transmission line flow limits (MVA), the bus voltage limits, and the real and reactive of generation limits are represented by a set of inequality constraints.

Minimize $\quad C = \sum C_i(P_{gi})$

Subject to:

$P_{Ti} + P_{di} - P_{gi} = 0$ [Energy balance constraints]

$Q_{Ti} + Q_{di} - Q_{gi} = 0$

$P_{gi,min} \le P_{gi} \le P_{gi,max}$ [Unit capacity constraints]

$Q_{gi,min} \le Q_{gi} \le Q_{gi,max}$

$|S_{ij}| \le S_{ij,max}$ [Branch flow constraints]

$V_{i,min} \le |V_i| \le V_{i,max}$ [Voltage constraints]

(3.23)

where

$$P_{Ti} = V_i^2 G_{ii} - V_i \sum_{m=1}^{n} V_m \left[G_{im}\mathrm{Cos}(\theta_i-\theta_m) + B_{im}\mathrm{Sin}(\theta_i-\theta_m)\right]$$

$$Q_{Ti} = -V_i^2 B_{ii} - V_i \sum_{m=1}^{n} V_m \left[G_{im}\mathrm{Sin}(\theta_i-\theta_m) - B_{im}\mathrm{Cos}(\theta_i-\theta_m)\right]$$

$$G_{ii} = \sum_{m=1}^{n} (G_{im} + G_{sim})$$

$$B_{ii} = \sum_{m=1}^{n} (B_{im} + B_{sim})$$

$$S_{ij} = P_{ij} + jQ_{ij}$$

$$P_{ij} = G_{ij}|V_i|^2 - G_{ij}|V_i||V_j|\,\mathrm{Cos}(\theta_i-\theta_j) - B_{ij}|V_i||V_j|\,\mathrm{Sin}(\theta_i-\theta_j)$$

$$Q_{ij} = -B_{ij}|V_i|^2 - G_{ij}|V_i||V_j|\,\mathrm{Sin}(\theta_i-\theta_j) + B_{ij}|V_i||V_j|\,\mathrm{Cos}(\theta_i-\theta_j)$$

C - Total system operating costs

C_i - Cost function of generating plant at bus i

C_{ij} - Cost function of the binding transmission line flow constraint

P_{gi} - Active power generation at bus i

Q_{gi} - Reactive power generation at bus i

P_{di} - Active power demand at bus i

Q_{di} - Reactive power demand at bus i

S_{ij} - Complex power flow from bus i to bus j

V_i - Voltage magnitude at bus i

θ_i - Voltage angle at bus i

$G_{im} + j\,B_{im}$ - Line admittance incident to bus i

$G_{sim} + j\,B_{sim}$ - Capacitive or inductive admittance at bus i

Note that Q_{gi}^{min} and Q_{gi}^{max} are functions of the active generation operating point whose relationship can be approximated using linearized functional curves. These relationships are shown in equations (3.24) and (3.25) respectively (Appendix A).

$$Q_{gi}^{min} = r + s\,P_{gi}^{op} \tag{3.24}$$

and

$$Q_{gi}^{max} = u + w\,P_{gi}^{op} \tag{3.25}$$

These values may also be obtained from the generator reactive capability curve if available. This OPF problem is solved using the AL algorithm previously described. Chapter 4 presents several examples to demonstrate this algorithm.

3.4. Parametric Analysis

Parametric analysis allows one to analyze system element variations with respect to their limits while maintaining the optimal solutions. In terms of optimization, this includes analyzing the changes in the cost vector, changes in the right-hand-side (RHS) vector, or the addition of new constraints. To maintain the optimality of the original solutions, these changes must be kept small.

For power interchange purposes, parametric analysis is used to investigate the acceptable range of variations, $\Delta P_{tie\text{-}bus}$, allowed at the boundary points. These variations represent the amount of interchange. Moreover, the variations made above or below the current generation level represent the amount of energy for sale or purchase respectively. Parametric analysis ensures that these variations will not cause any violations to the generating units, transformers, or transmission constraints. One advantage of parametric analysis is that the new optimal solutions can be determined without having to rerun another complete power flow problem.

3.4.1. Determination of the Amount of Energy for Sale

Generally, two neighboring systems can buy and sell any amount of power less than or equal to the transmission capabilities of the interconnection between them The transactions, however, also affect the power flows in the internal transmission system of the individual areas. Increases in flows caused by the scheduled interchange must be ensured not to overload any transmission lines. Therefore, the maximum amount of transaction energy must be evaluated with respect to the transmission constraints.

Consider the line flow constraint that is expressed as:

$$0 \leq P_{flow}{}^{op} + \Delta P_{flow} \leq P_{max,flow} \tag{3.26}$$

Let

$$\Delta P_{max,flow} = P_{max,flow} - P_{flow}{}^{op} \tag{3.27}$$

Then the upper constraint of equation (3 26) can be rewritten as

$$\Delta P_{flow} \leq \Delta P_{max,flow} \tag{3.28}$$

Equation (3.28) ensures that any consequences of the changes on the transmission line flows will satisfy the line capability limits. The relationship between the incremental line flow, ΔP_{flow}, and the incremental interchange at the tie buses, $\Delta P_{tie\text{-}bus}$, can be developed using the sensitivity matrix [H]. This is shown in equation (3.29).

$$\Delta P_{flow} = [H]\, \Delta P_{tie\text{-}bus} \tag{3.29}$$

The sensitivity matrix developed from the chain rule is shown in equation (3.30) which is evaluated at the present solution.

$$[H] = [\frac{\partial P_{flow}}{\partial P_{tie\text{-}bus}}] = [\frac{\partial P_{flow}}{\partial \theta} \quad \frac{\partial P_{flow}}{\partial v}] \begin{bmatrix} \dfrac{\partial \theta}{\partial P_{tie\text{-}bus}} \\ \dfrac{\partial V}{\partial P_{tie\text{-}bus}} \end{bmatrix} \tag{3.30}$$

The linearized power flow injection relationships are defined are defined as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P}{\partial \theta} & \dfrac{\partial P}{\partial v} \\ \dfrac{\partial Q}{\partial \theta} & \dfrac{\partial Q}{\partial v} \end{bmatrix}_{\theta_i, V_i} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \tag{3.31}$$

where θ_i and V_i are the present solution.

The derivative of each term of the Jacobian matrix in equation (3.31) is shown in Appendix B. The inverse relationship of equation (3.31) becomes

$$\begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \theta}{\partial P} & \dfrac{\partial \theta}{\partial Q} \\ \dfrac{\partial V}{\partial P} & \dfrac{\partial \theta}{\partial Q} \end{bmatrix}_{\theta_i, V_i} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{3.32}$$

The corresponding columns of the inverse of the Jacobian matrix in equation (3.32) represent the elements of vector $\partial\theta/\partial P_{tie\text{-}bus}$ and $\partial V/\partial P_{tie\text{-}bus}$ in equation (3 30). Their values can be calculated by using forward elimination and backward substitution.

Together the elements of $\partial P_{flow}/\partial\theta$ and $\partial P_{flow}/\partial V$ in equation (3 30) will have a maximum of four nonzero elements for every power flow from bus i to bus j. These are $\partial P_{ij}/\partial\theta_i$, $\partial P_{ij}/\partial\theta_j$, $\partial P_{ij}/\partial V_i$, and $\partial P_{ij}/\partial V_j$. They are shown in equations (3.33) - (3.36) respectively [15].

$$\frac{\partial P_{ij}}{\partial\theta_i} = - V_i V_j Y_{ij} \sin(\theta_i-\theta_j-\alpha_{ij}) \qquad (3\ 33)$$

$$\frac{\partial P_{ij}}{\partial\theta_j} = V_i V_j Y_{ij} \sin(\theta_i-\theta_j-\alpha_{ij}) \qquad (3.34)$$

$$\frac{\partial P_{ij}}{\partial V_i} = V_j Y_{ij} \cos(\theta_i-\theta_j-\alpha_{ij}) - 2 V_i (Y_{ij} \cos \alpha_{ij}) \qquad (3.35)$$

$$\frac{\partial P_{ij}}{\partial V_j} = V_i Y_{ij} \cos(\theta_i-\theta_j-\alpha_{ij}) \qquad (3.36)$$

Once the sensitivity matrix [H] is formed, equation (3.28) can be expanded as

$$[H]\ \Delta P_{tie\text{-}bus} \le \Delta P_{max,flow} \qquad (3.37)$$

It is important to note that if a load bus is chosen as the slack bus, then it cannot be used as an interchange bus.

3.4.2. Determination of the Amount of Energy for Purchase

The same procedure can be repeated to find the amount of energy for purchase. From equations (3.26), the lower bound is expressed as

$$- P_{flow}{}^{op} \leq \Delta P_{flow} \tag{3.38}$$

Define $\Delta P_{min,flow}$ as $P_{flow}{}^{op}$, then equation (3.38) can be rewritten as

$$\Delta P_{flow} \leq \Delta P_{min,flow} \tag{3.39}$$

Equation (3.39) guarantees that the line power flow does not change direction, thus satisfying the optimality conditions. Analogous to equation (3.37), the relationship between the decremental power flow and the decremental power interchange at the tie-bus is expressed as

$$[H]\ \Delta P_{tie\text{-}bus} \leq \Delta P_{min,flow} \tag{3.40}$$

where [H] is the same sensitivity matrix previously described. Once calculated, the amount of interchange energy and the price quotations are transferred to the central broker. The broker then sets up the transactions accordingly.

It is often desirable to know how much the total operating costs will change when the new interchange schedules are implemented. Equation (3.41) provides an estimate to the incremental operating cost because of interchange

$$\Delta F = \sum_{i \in \text{tie-bus}} \rho_{pi}^{*} \, \Delta P_i \tag{3.41}$$

For the selling utility, ΔF reflects the amount of profits that the company should make to cover the extra cost of generation. Similarly, for the purchasing utility, ΔF reflects the amount of the cost of generation that the company can avoid.

3 4.3. Determination of the Participation Factor of each Generator

Questions often arise to as how much each generating unit should be shifted to serve the new load level in the most economical way. As the system load level changes by the amount of $\Delta P_{\text{tie-bus}}$, the participating generators must also respond correspondingly such that

$$\Delta P_{\text{tie-bus}} = \Delta P_{G1} + \Delta P_{G2} + \ldots + \Delta P_{GN} \tag{3.42}$$

Assume that both the first and the second derivatives in the cost versus power output function are available i.e. $F'(P_{Gi}^{op})$ and $F''(P_{Gi}^{op})$ exist. The incremental cost curve of the i^{th} unit is shown in Figure 3.2.

When the load level changes, the i^{th} generator output shifts by the amount of ΔP_{Gi}. Consequently, the system incremental cost also moves from λ^{op} to $\lambda^{op} + \Delta\lambda$. For a small change in ΔP_{Gi}, $\Delta\lambda_i$ can be approximated as shown in equation (3.43).

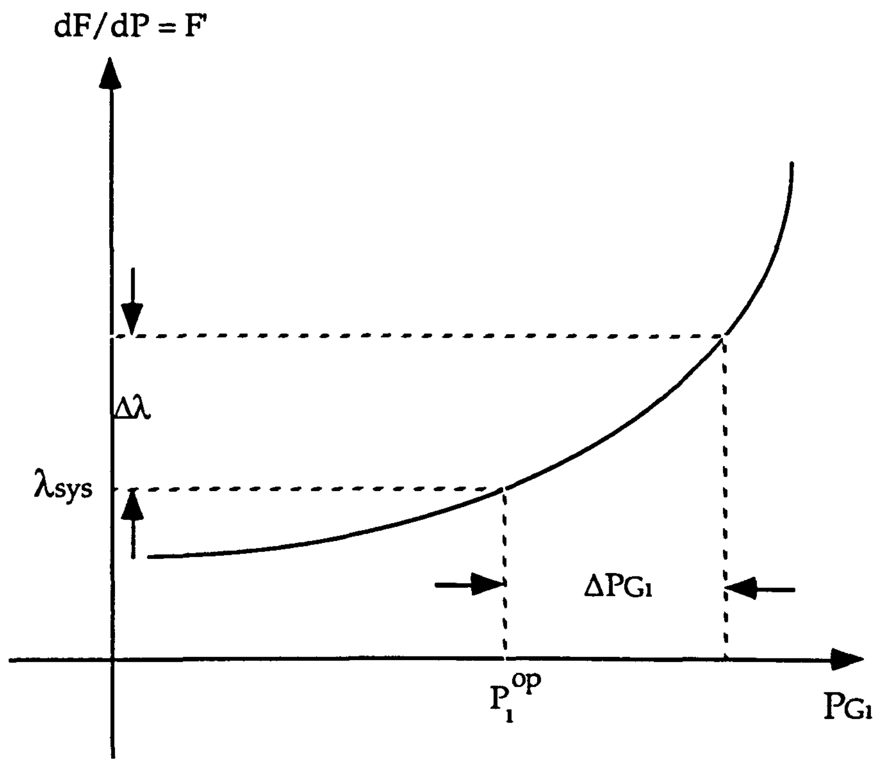

Figure 3.2. Incremental cost curve of P_{Gi}.

$$\Delta\lambda_i = \Delta\lambda_{sys} \approx F''(P_{Gi}{}^{op})\,\Delta P_{Gi} \tag{3.43}$$

or

$$\Delta P_{Gi} = \Delta\lambda_{sys} \,/\, F''(P_{Gi}{}^{op}) \tag{3.44}$$

Substituting the expression for ΔP_{Gi} from equation (3.44) into equation (3 42), $\Delta P_{tie\text{-}bus}$ can be written as

$$\Delta P_{tie\text{-}bus} = \Delta\lambda_{sys} \sum_{i=1}^{N} \left(\frac{1}{F''_{P_{Gi}}}\right) \tag{3.45}$$

Substituting equation (3.43) into (3.45), and upon simplification, the participation factor for each unit is expressed as

$$\left(\frac{\Delta P_{Gi}}{\Delta P_{tie\text{-}bus}}\right) = \frac{\left(\dfrac{1}{F''_{P_{Gi}}}\right)}{\sum_{i}^{N}\left(\dfrac{1}{F''_{P_{Gi}}}\right)} \tag{3.46}$$

This factor multiplied by the amount of interchange represents the increment for generator i. The new generation level for unit i is the summation of the original operating level and the increment.

The participation factor calculation (PFC) method is a major improvement over the classical approach. The additional power export/import is no longer absorbed by the slack generator. Instead, it is distributed among all the on-line generating units so that the load is continually served in the most economical way.

3.5. Incorporation of Regulating Transformer in OPF Calculation

Phase-shifting and tap-changing-under-load (TCUL) transformers have been used extensively in modern power system to regulate the line real power flows and the PQ-bus voltage magnitudes, respectively; their purpose is to enhance the power system security and minimize system losses [9].

Figure 3.3 depicts a regulating transformer connected between bus i and bus j. The transformer is assumed to have both the phase-shifting and the tap-changing capabilities with phase-shift angle ϕ_{ij} and tap setting t_{ij}. The phase-shift angle is adjusted to regulate the real power flows. The tap setting is used to control the P-Q bus voltage magnitudes. The transformer tap side is connected to bus i.

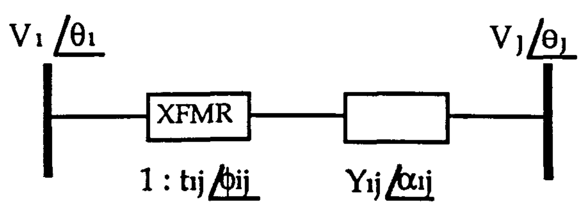

Figure 3.3. Regulating transformer diagram

Let

$$\alpha_{ij} = \text{Arctan}(B/G) \tag{3.47}$$

and

$$|Y_{ij}| = |G + jB| = |Y_{ij} \text{Cos}\,\alpha_{ij} + Y_{ij} \text{Sin}\,\alpha_{ij}| \tag{3.48}$$

The line power flow equation at the transformer termination buses can be written as [9]

$$P_{ij}^{reg} = V_i^2\, t_{ij}^2\, G_{ij} - V_i\, V_j\, Y_{ij}\, t_{ij}\, \text{Cos}(\theta_i - \theta_j + \phi_{ij} - \alpha_{ij}) \tag{3 49}$$

$$Q_{ij}^{reg} = V_i^2\, t_{ij}^2\, (B_{ij} - b_{sh,ij}/2) - V_i\, V_j\, Y_{ij}\, t_{ij}\, \text{Sin}(\theta_i - \theta_j + \phi_{ij} - \alpha_{ij}) \tag{3.50}$$

$$P_{ji}^{reg} = V_j^2\, G_{ij} - V_i\, V_j\, Y_{ij}\, t_{ij}\, \text{Cos}(\theta_j - \theta_i + \phi_{ij} - \alpha_{ij}) \tag{3 51}$$

$$Q_{ji}^{reg} = V_j^2\, (B_{ij} - b_{sh,ij}/2) - V_i\, V_j\, Y_{ij}\, t_{ij}\, \text{Sin}(\theta_j - \theta_i + \phi_{ij} - \alpha_{ij}) \tag{3.52}$$

Note that equations (3.49) and (3.50) are asymmetrical with equations (3.51) and (3.52), respectively. Conversely, when the equations do not include the transformer representations, then equations (3.49) and (3.50) become symmetrical with equations (3.51) and (3.52). Either pair of equations can be used to describe the power flows through a transmission line. Without transformer representations ($t_{ij} = 1$ and $\phi_{ij} = 0$), the line power flow can simply be described by equations (3.53) and (3.54).

$$P_{ij} = G|V_i|^2 - G_{ij}|V_i||V_j| \, Cos(\theta_i - \theta_j) - B_{ij}|V_i||V_j| \, Sin(\theta_i - \theta_j) \quad (3.53)$$

$$Q_{ij} = -B|V_i|^2 - G_{ij}|V_i||V_j| \, Sin(\theta_i - \theta_j) + B_{ij}|V_i||V_j| \, Cos(\theta_i - \theta_j) \quad (3.54)$$

The real and reactive power injections into a bus are the sum of the transmission flows and the transformer branches incident to that bus.

$$P_{Ti} = \sum P_{ij} + P_{ij}^{reg} \quad (3.55)$$

$$Q_{Ti} = \sum Q_{ij} + Q_{ij}^{reg} \quad (3.56)$$

To include the TCUL transformer in the OPF formulation, first P_{ij}^{reg}, Q_{ij}^{reg}, P_{ij}^{reg}, and Q_{ij}^{reg} are calculated using equations (3.49)-(3.52) respectively. Then the expressions for P_{Ti} and Q_{Ti} in equation (3.23) are also replaced with equations (3.55) and (3.56) respectively. An equality constraint is added to the original problem formulation to limit the line's real power flow to P^{spec}. Equation (3 57) shows this constraint where t_{ij} is assumed to have a value of one.

$$P_{ij}^{spec} - V_i^2 G_{ij} - V_i V_j Y_{ij} \, Cos(\theta_i - \theta_j + \phi_{ij} - \alpha_{ij}) = 0 \quad (3.57)$$

The OPF problem is solved to obtain the phase shift angle ϕ_{ij}.

3.6. Economic Interpretation of the Lagrangian Multiplier and Determination of the Short-Run Marginal Costs

In the real world problem, the decision variables usually represent certain physical quantities. Similarly, the Lagrangian multiplier often has an economic meaning that can provide valuable information to the decision maker. The specific interpretation of this multiplier may vary depending on what units they represent. For instance, a power system planner may be interested to know how much energy (MW) each generator should provide to minimize the per-unit cost of electricity. This is also called the economic dispatch calculation (EDC) problem.

Let the equality constraints represent the demand to be satisfied. For simplicity, the inequality constraints are neglected. The primal problem can be formulated as

$$\text{Minimize} \quad C_i\ (\$/\text{MW}) \times P_{Gi}\ (\text{MW})$$

$$\text{Subject to}$$

$$[D_{bus\ i}\ (\text{MW})\ \text{for each bus in the system}] \qquad (3.58)$$

and its dual form is

Maximize $D_{bus\,i}$ (MW) x $\rho_{p\,i}$ (\$/MW)

Subject to

[Cost activities associated with each generator (\$/MW)]

(3.59)

The dual form is interpreted as follows. Instead of trying to obtain the most desirable mix of generator outputs, the system planner would like to calculate a price set that maximizes the return of operating costs. Therefore, an equilibrium set of activities and a set of prices exist where the minimal production cost is equal to the maximal return.

The Lagrangian multiplier is the marginal cost of providing one unit of electricity to the customer. Economically, it is the fair price that the customer should pay for an extra unit of electricity. By definition, the Lagrangian multipliers are the short-run marginal prices or the spot prices. The Lagrangian function for the above OPF formulation is shown in equation (3.60).

$$
\begin{aligned}
L\,(Pg, Qg, V, \theta) = & \\
& \sum \lambda_i\, C_i(P_{gi}) \\
& + \sum \rho_{pi}\,(P_{Ti} + P_{di} - P_{gi}) \\
& + \sum \rho_{qi}\,(Q_{Ti} + Q_{di} - Q_{gi})
\end{aligned}
$$

$$
\begin{aligned}
&+ \sum \eta_{i,min} (P_{gi,min} - P_{gi}) \\
&+ \sum \eta_{i,max} (P_{gi} - P_{gi,max}) \\
&+ \sum \mu_{i,min} (Q_{gi,min} - Q_{gi}) \\
&+ \sum \mu_{i,max} (Q_{gi} - Q_{gi,max}) \\
&+ \sum \sum \tau_{ij} \; (|S_{ij}| - S_{ij,max}) \\
&+ \sum \nu_{i,min} (V_{i,min} - |V_i|) \\
&+ \sum \nu_{i,max} (|V_i| - V_{i,max})
\end{aligned}
\tag{3 60}
$$

where

λ_i - Marginal operating cost at bus i

ρ_{pi} - The Lagrangian multiplier for the active power equation at bus i

ρ_{qi} - The Lagrangian multiplier for the reactive power equation at bus i

$\eta_{i,min}$ - The Lagrangian multiplier for the minimum active power generation limit at bus i

$\eta_{i,max}$ - The Lagrangian multiplier for the maximum active power generation limit at bus i

$\mu_{i,min}$ - The Lagrangian multiplier for the minimum reactive power generation limit at bus i

$\mu_{i,max}$ - The Lagrangian multiplier for the maximum reactive power generation limit at bus i

τ_{ij} - The Lagrangian multiplier for the complex power flow limit from bus i to bus j

$\nu_{i,min}$ - The Lagrangian multiplier for the minimum voltage level at bus i

$\nu_{i,max}$ - The Lagrangian multiplier for the maximum voltage level at bus i

According to the spot pricing theorem [10], the real-time prices of active and reactive power at bus i at a particular time are given by

$$\pi_{pi} = \frac{\delta}{\delta Pd_i} [\text{ Total cost of providing electricity to all customers subject to operational constraints }] \tag{3.61}$$

and

$$\pi_{qi} = \frac{\delta}{\delta Qd_i} [\text{ Total cost of providing electricity to all customers subject to operational constraints }] \tag{3.62}$$

Equations (3.61) and (3.62) are the first-order derivatives of the Lagrangian function with respect to the real and reactive power demands. Taking the first derivatives of the Lagrangian function, the spot prices at bus i, π_{pi} and π_{qi}, are expressed as:

$$\pi_{pi} = \frac{\delta L}{\delta Pd_i} = \rho_{pi} \tag{3.63}$$

$$\pi_{qi} = \frac{\delta L}{\delta Qd_i} = \rho_{qi} \tag{3.64}$$

Equations (3.63) and (3.64) show that the spot prices, π_{pi} and π_{qi}, are the Lagrangian multipliers of the OPF problem. Therefore, the optimal multipliers from the OPF solution automatically determine the real-time prices for every bus.

Define the system lambda, λ_{sys}, as the short-run marginal generating cost. Specifically, it is the cost of transporting another KWh of energy from a marginal unit despite the losses and the transmission constraints. This lambda is also recognized as the optimal spot price at a reference bus, called the slack bus [10]. The system lambda is the same at every bus. However, it may not be optimal at another location other than the slack bus. To obtain the optimal spot price at bus i, the system lambda must be multiplied by the incremental transmission loss term. This term varies for different locations, depending upon the network losses and the transmission line constraints. If an increase in demand causes a larger increase in system loss, then the customers at that location experience higher spot prices. Conversely, an increase in demand that incurs a smaller loss obtain lower spot prices.

Recall that the spot price is composed of the marginal generation cost, the marginal network losses, the marginal quality of supply, and the revenue reconciliation. The quality of the supply charge arises only when operating limits are being approached. It can be shown that these effects do not happen accidentally.

According to the KKT optimality conditions,

$$\frac{\delta L}{\delta P_{g_i}} = \lambda_i - \rho_{pi} - \eta_{i,min} + \eta_{i,max} = 0 \tag{3.65}$$

and

$$\frac{\delta L}{\delta Q_{g_i}} = - \rho_{qi} - \mu_{i,min} + \mu_{i,max} = 0 \tag{3.66}$$

The spot prices of the active and reactive power at the generating bus can be expressed as

$$\rho_{pi} = \lambda_i - \eta_{i,min} + \eta_{i,max} \tag{3.67}$$

$$\rho_{qi} = -\mu_{i,min} + \mu_{i,max} \tag{3.68}$$

The multipliers, η and μ, are recognized as the shadow values of the generating capacity. They represent the charges necessary to avoid excessive demand These values are zero unless the generator at bus i is fully loaded.

Figure 3.4 illustrates the spot pricing phenomena. First, the most efficient unit, the generator at bus one, is dispatched to satisfy the system demand. As load increases, unit one must raise its output to satisfy the new system load and losses. This is followed by the increase of the marginal cost along unit one's incremental cost curve. The marginal cost will continue to rise until the generating capacity limit is reached. After that, it rises vertically by the amount of $\eta_{1,max}$.

By analogy, the spot price of the reactive power at any generating bus i is zero until the reactive generation capacity limit is reached. Moreover, the price for reactive power varies with the system losses as the load changes. If an increase in the reactive power demand at a bus reduces the system losses, it is then possible to have a negative price for reactive power at that bus.

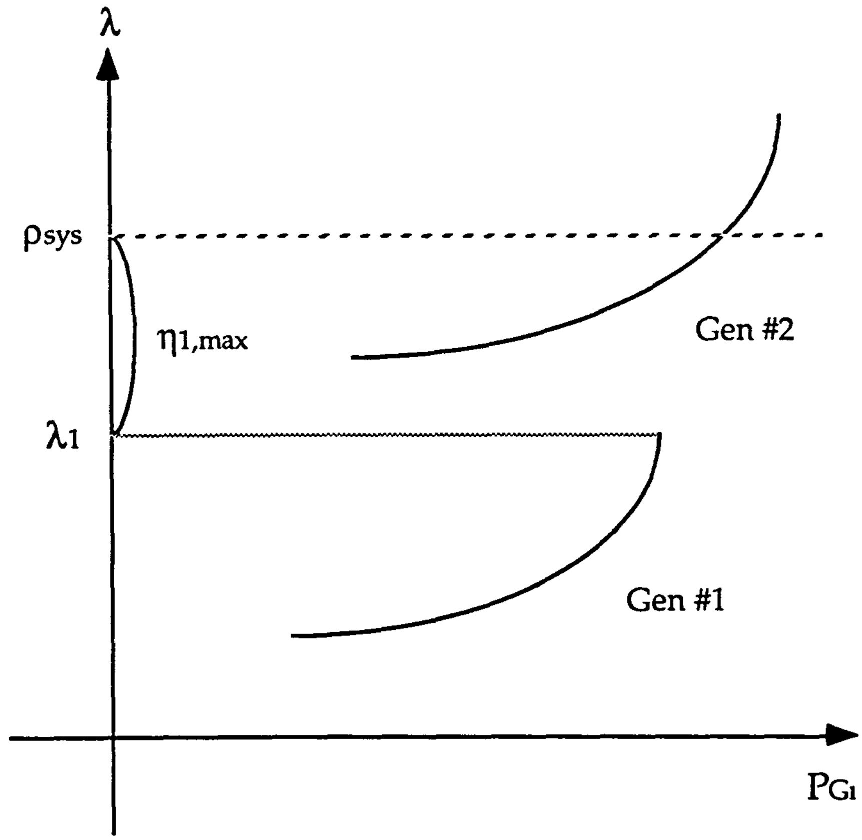

Figure 3.4. Incremental cost curve of a two-generator system

Economically, this means that the utility would prefer to pay the customer to consume more reactive power at that bus since this will reduce the total system losses.

Transmission limits also effect the real-time prices at each bus. When the power flow on a line is tightened, the real-time prices at the receiving end increase. This happens because the line flow constraint has forced the use of other higher loss paths to satisfy the demand requirements at the receiving end of the line. As a result, more expensive generators may have to be dispatched to satisfy the loads that in turn increase the total operating cost.

In addition, the system voltage must be maintained within an acceptable range so that real power can be supplied to the customers. Low voltage results in poor quality of service i.e. dim lights and overheated motors. In more severe cases, it may cause motors to drop off line, voltage collapse, or even a blackout. High voltage results in shorter life, inefficient operation, and damage to the electric equipment. When bus voltages become stressed, the local high prices will cause the customers to lower their reactive power demand consumption.

3.7. Computerized Brokerage Operation

The interchange brokerage process described in Chapter 2 is implemented using LP optimization to maximize the profits for the total system. The LP formulation is given in equation (3.69). The objective function maximizes the total net savings obtained from a possible interchange scheduled between area i and j. The right-hand-side (RHS) of the inequality constraint set represents the maximum amount of energy allowed to be exported or imported from a given area. The amount of energy for interchange is determined from the parametric analysis discussed earlier. The LP formulation for the brokerage operation is presented in equation (3.69).

$$\text{Maximize} \quad \sum (\lambda_{bj} - \lambda_{si})\, I_{ij}$$

Subject to:

$$I_{12} + I_{13} + \dots + I_{1m} \le I_1^{s}$$

$$I_{21} + I_{23} + \dots + I_{2m} \le I_2^{s}$$

$$\vdots$$

$$I_{n1} + I_{n2} + \dots + I_{nm} \le I_n^{s}$$

$$I_{21} + I_{31} + \dots + I_{m1} \le I_1^{b}$$

$$I_{12} + I_{32} + \dots + I_{m2} \le I_2^{b}$$

$$\vdots$$

$$I_{1n} + I_{2n} + \dots + I_{mn} \le I_n^{b}$$

$$0 \le I_{ij} \le I_{ij}^{max}, \quad i \ne j \tag{3.69}$$

Where

s - Subscript representing selling area

b - Subscript representing buying area

I_{ij} - The amount of interchange between selling and buying area

I_{ij}^{max} - Tie-line capability between selling and buying area

λ_{bj} - Decremental cost of buying area ($/MWh)

λ_{si} - Incremental cost of selling area ($/MWh)

I^b - The maximum amount of energy sold to the buying area

I^s - The maximum amount of energy purchased from the selling area

The price quotations are obtained from the OPF solution at a given operating point; these quotations represent the spot prices at the interchange buses. Most transactions use the system incremental cost or the marginal generating cost of a unit as the price quotation. Practically speaking, this does not reflect the true value. Additional costs because of the network losses and the transmission constraints may be incurred in transporting energy from the generating stations to the delivery points. Moreover, an increase in load because of interchange activities also may stress the generation or the network system Spot prices indicate the true economic values of the transaction costs. When the generation or the network limits are stressed, the shadow prices become active. These shadow prices are reflected in the spot prices.

If a wheeling arrangement exists, both the wheeling and the loss charges must be considered before matching buyers and sellers. These are defined in equations (3.70) and (3.71) respectively.

$$\text{Wheeling Charge} = FC \times I_{ikj} \tag{3.70}$$

$$\text{Loss Charge} = (\lambda_{bpt} - \lambda_{spt})\, I_{ikj} \tag{3.71}$$

I_{ikj} represents the amount of energy transferred from area i to area j through an intermediate area k. The notation "spt" reflects the boundary points of the local control centers between area s and area w. The notation "bpt" reflects the boundary points between area w and area b FC is the fixed charge (in $/MWh) for revenue reconciliation purposes and may vary depending on the network configuration and the method used as well. For instance, the Florida brokerage system has used a $ 1.00/MWh figure for its fixed charge [16].

The loss charge calculation is based on the marginal cost concept [10] It is calculated as the difference between the optimal spot prices at buses spt and bpt, multiplied by the total amount of energy that is wheeled through the network [10]. The total charge, TC, is the summation of the loss and wheeling charges, and is shown in equation (3.72).

$$TC = [FC + (\lambda_{bpt} - \lambda_{spt})] \times I_{ikj} \tag{3.72}$$

The net saving (NS) calculation is given in equation (3.73).

$$NS = (\lambda_{bj} - \lambda_{si})\, I_{ij} - TC \tag{3.73}$$

The LP formulation for the interchange brokering can be extended easily to include wheeling transactions [17]. This is shown in equation (3.74).

Maximize $\sum [(\lambda_{bj} - \lambda_{si}) I_{ij} - TC]$

Subject to:

$$I_{ij} + I_{ikj} \leq I_i^{s}$$

$$I_{ji} + I_{jki} \leq I_j^{s}$$

$$\cdot$$

$$\cdot$$

$$I_{ij} + I_{ikj} \leq I_j^{p}$$

$$I_{ji} + I_{jki} \leq I_i^{p}$$

$$0 \leq I_{ikj} + I_{ik} \leq I_{ikj}^{max}, \quad i \neq j \neq k$$

$$0 \leq I_{ikj} + I_{kj} \leq I_{ikj}^{max}, \quad i \neq j \neq k$$

(3.74)

CHAPTER IV. RESULTS

A computer program implementing the AL optimization technique was created to test the OPF. Several cases were used and the results are summarized. The interchange brokerage transaction was implemented using the GAMS/MINOS commercial LP software package. The test cases were presented as follows:

[i] Economic Dispatch Calculation (EDC).

[ii] Optimal Power Flow.

[iii] OPF with the Inclusion of Regulating Transformer.

[iv] Parametric Analysis.

[v] Brokerage Interchange Transaction for a Four-Area System.

[vi] Effect of Wheeling Transactions.

All cases assumed a steady-state operating condition during the brokerage interchange study. The first example illustrated an EDC using a 2-generator system. Example two illustrated the use of the AL OPF technique to minimize the operating cost and eliminate line overloading. The third example examined the effects of including a regulating transformer to control the real power flow The parametric analysis was performed using example 4. It was used to determine the maximum amount of interchange energy with respect to the generating units or transmission constraints. The initial operating system was assumed to have adequate capacity

Examples five and six showed the brokerage transactions. A four-area interconnected network served to illustrate the brokerage implementation. Example five examined the brokerage interchange without wheeling and example six analyzed the effect of wheeling transactions. The transactions were set up as an LP model and were solved using GAMS/MINOS. Both results were examined and the savings were compared.

4.1. Example 1: Economic Dispatch Calculation

Table 4.1 shows the data for a 2-generator unit system. Both units were dispatched to satisfy a total demand of 175 MW. The objective function was to minimize the total operating costs The constrained problem was formulated as shown in equation (4.1).

The AL minimization technique was implemented to solve the economic dispatch problem. The problem was solved in three iterations and the results are shown in Table 4 2. All constraints were satisfied; the total output met the desired 175 MW demand. Note that infeasible initial parameter values were chosen initially This demonstrates that AL can handle infeasible starting points.

Table 4.1. Generator data

Unit#	Fuel Type	Max Output (MW)	Min Output (MW)	Fuel Cost ($/MBtu)	I/O Curve $[a+bP+dP^2]$ a, b, d
1	Coal	160	80	1.0	100.267 -29.745 6.7252
2	Oil	80	70	1.0	40.8915 2.8571 6.065

Minimize $\sum F_{PG_i}$

Subject to

$$\sum P_{G_i} = 175.0 \qquad\qquad i = 1,2$$

$$80 \leq P_{G1} \leq 160$$

$$50 \leq P_{G2} \leq 100$$

where

$$F_{PG1} = H_1 \times 1.0 = 100.267 - 29.745\, P_{G1} + 6.7252\, P_{G1}^2 \quad \$/h$$

$$F_{PG2} = H_2 \times 1.0 = 40.8915 + 2.8571\, P_{G2} + 6.065\, P_{G2}^2 \quad \$/h \tag{4.1}$$

Table 4.2. Economic dispatch results

Tolerance (Eps1)[a]	1.0E-04	
Tolerance (Eps2)[b]	1.0E-10	
Initial Penalty Factors	5 0	
Initial Parameter Values (P_{G1}, P_{G2})	0.0	0.0

~~~~~ P R I N T - R E S U L T S ~~~~~

Objective Function	$ 150.42	
P_{G1} P_{G2} (MW)	105.78	69.22
Lambda ($/MWh)	0.1126	
Penalty Factors	45.0	45 0
No of Iterations	3	

a Eps1 is the tolerance for the dual maximization

b Eps2 is the tolerance for the augmented Lagrangian minimization
~~~~~

4.2. Example 2: Optimal Power Flow

Figure 4.1 shows a four-bus network example. This system was used to test the performance of the augmented Lagrangian OPF. This network consisted of two generator buses and two load buses. Bus 4 served as the reference bus. No transformers, phase-shifters, or series capacitor banks were included for simplicity. In addition, reactive generating limits were ignored. The generator and the transmission line data were taken from reference [17].

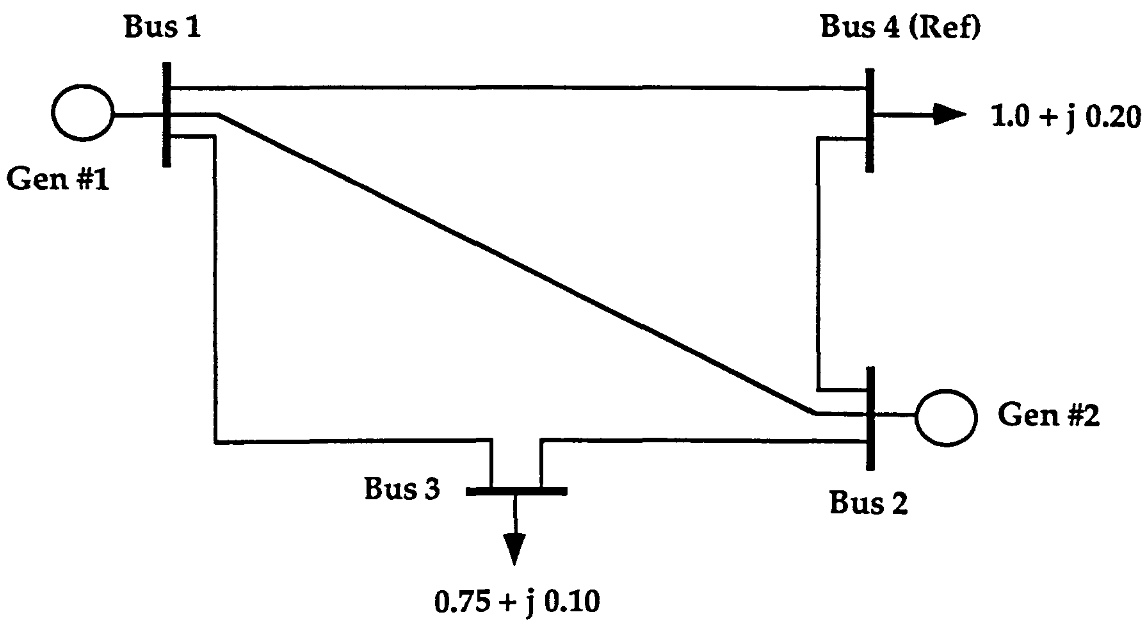

Figure 4 1. A four-bus network for example 3

The generator cost function data was taken from the previous example. Table 4.3 and Table 4.4 gives the initial generation data and load data respectively. The transmission line data is shown in Table 4.5. The actual power flow operating conditions were first analyzed prior to optimizing the system. They were obtained from the power flow program solutions [17]. The results are shown in Table 4.6 and Table 4.7 respectively.

Table 4.3. Generation bus data

Bus #	P	Q	V_{lower}	V_{upper}
1	1.26	0.200	0.90	1.10
2	0.52	0.262	0.90	1.10

Table 4.4. Load bus data

Bus #	P	Q	V_{lower}	V_{upper}
3	0.75	0.10	0.95	1.05
4	1.00	0.20	1.00	1.00

Table 4.5. Transmission line data

Line #	I_{bus}	J_{bus}	G_{pu}	B_{pu}	S_{rated}
1	1	2	0.48070	- 2.40380	0.0700
2	1	3	1.92308	- 9.61538	0.6021
3	1	4	0.96150	- 4.80770	0.7280
4	2	3	0.38461	- 1.92300	0.2236
5	2	4	0.76923	- 3.84620	0.5590

Table 4 6. Original bus voltage data

Voltage	Angle (degrees)
1.0440	6.1298
1.0570	5.4317
1.0241	2.7572
1.0000	0.0000

Table 4.7 Original line flow data

Line #	I_{bus}	J_{bus}	P_{ij}	Q_{ij}	S_{ij}
1	1	2	0.026	- 0.039	0.0469
2	1	3	0.648	0.096	0.6551
3	1	4	0.586	0.142	0.6029
4	2	3	0.111	0.050	0.1217
5	2	4	0 435	0.173	0.4681

The initial power flow results from Table 4.7 were compared with the line rating requirements. Overloading occurred on line 1-3. The rated $S_{1\text{-}3}$ was required to be 0 6021 pu; instead it was 0.6551 pu.

The OPF problem was formulated as described in equation (3.23). The objective function was to minimize the total operating costs subject to the real power generation capability limits, real and reactive power transmission line limits, and bus voltage constraints. The OPF converged in seven iterations. The overload problem was alleviated. Table 4.8 shows the results.

Initially, the total system losses were 0.03 pu and increase to 0.0332 pu This occurred because the transmission constraint on line 1-3 had forced the excessive power to flow through other higher loss paths to satisfy the load. This resulted in generation redispatch and increased the total operating costs. The initial cost was $150 79 and increased to $ 150.80.

The spot prices for the real and reactive power at each bus were obtained directly from the values of the optimal Lagrangian multipliers ρ_{pi} and ρ_{qi} respectively. They are shown in Table 4.8.

4.3. Example 3: OPF with the Inclusion of a Regulating Transformer

Since line 1-3 was most likely to become overloaded, a phase-shifting transformer was used to regulate the real power flow on line 1-3. The transformer tap side was connected to bus 1. From example 4.2, the active power flow on line 1-3 was 0.595 pu. For reliability purposes, it was desired to

Table 4.8. OPF results of example 4.2.

Tolerance (Eps1) 1.0E-06

Tolerance (Eps2) 1.0E-10

Initial Penalty Factors 59 3728

Initial Parameter Values

$P_{G1}, Q_{G1}, P_{G2}, Q_{G2}$ 1.0 0.0 0 8 0.0

$|V_1|$ $|V_2|$ $|V_3|$ 1.0 1.0 1.0

Bus Voltage Angles 1, 2, 3 (in radians) 0 0 0.0 0.0

~~~~~ P R I N T - R E S U L T S ~~~~~

Objective Function* ($) 150 80

P_{G1} Q_{G1} (in p.u.) 1.058 0.183

P_{G2} Q_{G2} (in p.u.) 0.725 0.282

$|V_1|$ $|V_2|$ $|V_3|$ (in p.u.) 1.04096 1.06109 1.0222

Bus Voltage Angles 1, 2, 3 (in radians) 0 09434 0.11109 0.04012

Slack Bus (Bus #4)
$|V4|$: 1.0 p.u.
Angle (In radians): 0.0
~~~~~

Table 4.8 (continued)

Active & Reactive Line Flows (in p.u)

Line #	P_{ij}	Q_{ij}	S_{ij}
1-2	- 0.053919	- 0.0412269	- 0.067875
1-3	0.595067	0.0917531	0.602099
1-4	0.516912	0.1329821	0.533743
2-3	0 164393	0.0550478	0.173365
2-4	0.506502	0.1840845	0.538916

Spot - Prices ($ / MWh)

Bus #	1	2	3	4
ρ_{pi}	0.112568	0.116509	0.132100	0.119338
ρ_{qi}	0.000012	0.000002	0.003078	0.000696

Penalty Factors	4500.0	4500.0	4500.0
No of Iterations	7		

maintain P_{1-3} at 0.55 pu. Then, the phase-shift angle of the transformer was adjusted. The tap setting was set to unity. The phase-shift angle was treated as the unknown variable and was included in the OPF formulation using equations (3.49) - (3.52). The problem was solved using the AL OPF software The results are shown in Table 4 9. Note that the inclusion of the phase shifter has reduced the total system losses (0.0329 pu). The total operating costs were also reduced as a result. Table 4.9 shows that the total operating costs decreased from $150 801 to $150.795.

4.4. Example 4: Parametric Analysis

Figure 4.2 shows the 4-bus system from previous example with P_{load} at bus 3 was changed to 0.50 pu. Bus number 3 was treated as the interchange bus. In addition, no transformers, phase shifters, or series capacitor banks were included. To perform a parametric analysis, the initial operating conditions were first determined. The results were obtained from the OPF solutions. These are shown in Table 4.10.

4.4.1. Determination of the Amount of Interchange

This example shows how to use parametric analysis to determine the amount of energy for interchange. First, the linearized sensitivity relationship between transmission line limit and the power flow injection

Table 4.9. OPF results of example 4.3

Tolerance (Eps1)	1.0E-06			
Tolerance (Eps2)	1 0E-10			
Initial Penalty Factors	5.0			

<u>Initial Parameter Values</u>

P_{G1}, Q_{G1}, P_{G2}, Q_{G2}	1.0	0.0	0.8	0.0
$\lvert V_1 \rvert$ $\lvert V_2 \rvert$ $\lvert V_3 \rvert$	1.0	1 0	1.0	
Bus Voltage Angles 1, 2, 3 (in radians)	0.0	0.0	0.0	

~~~~~ P R I N T - R E S U L T S ~~~~~

Objective Function* ($)	150.79		
P_{G1} Q_{G1} (in p.u.)	1.0829	0.2969	
P_{G2} Q_{G2} (in p.u.)	0.7080	0.1685	
$\lvert V_1 \rvert$ $\lvert V_2 \rvert$ $\lvert V_3 \rvert$ (in p.u.)	1.0486	1.0515	1.0269
Bus Voltage Angles 1, 2, 3 (in radians)	0.09817	0.10627	0.01078

<u>Slack Bus (Bus #4)</u>

$\lvert V4 \rvert$ 1.0 p.u.

Angle (In radians) 0.0
~~~~~

Table 4.9 (continued)

Regulating Line (Line #1-3) $|P_{spec}| = 0.50$ p.u.

Phase-Shift Angle of XFMR connecting Line 1-3

ϕ_{13} (in degree) = - 2.2165

Active & Reactive Line Flows (in p.u.)

Line #	P_{ij}	Q_{ij}	S_{ij}
1-2	- 0.022972	- 0.003155	- 0.023187
1-3	**0.550000**	**0.129489**	**0.565037**
1-4	0.547884	0.170235	0.565037
2-3	0.209824	0.019633	0.210741
2-4	0.475261	0.145494	0.497033

Spot - Prices ($ / MWh)

Bus #	1	2	3	4
ρ_{pi}	0.1148254	0.1144614	0.1190910	0.11969789
ρ_{qi}	0.0000092	0.0000034	0.0095691	0.00076705

Penalty Factors 4500 0 4500.0 4500.0

No of Iterations 8

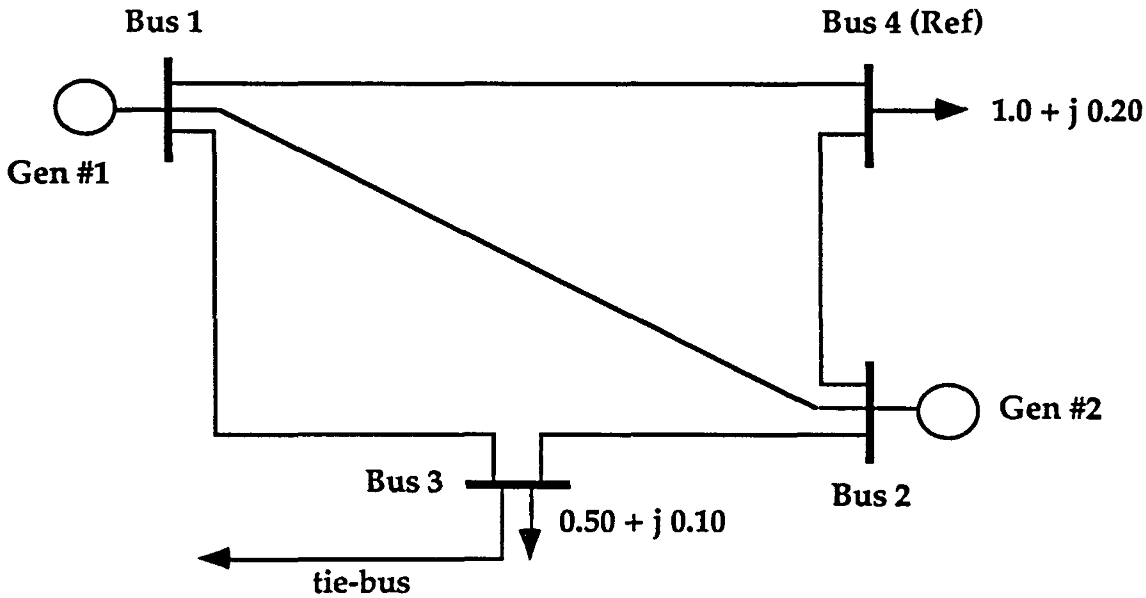

Figure 4.2. A four-bus system for example 4

Table 4.10. OPF results with P_{load} at bus 3 equals to 0.50 pu

Objective Function* ($)	148.07		
P_{G1} Q_{G1} (in p.u.)	0.9530	0.2777	
P_{G2} Q_{G2} (in p.u.)	0.5743	0.1583	
$\lvert V_1 \rvert$ $\lvert V_2 \rvert$ $\lvert V_3 \rvert$ (in p.u.)	1.04965	1.05016	1.03288
Bus Voltage Angles 1, 2, 3 (in Radians)	0.09946	0.104664	0.06342

Table 4.10 (continued)

Slack Bus (Bus #4)

|V4| 1.0 p.u.

Angle (In radians) 0.0

Active & Reactive Line Flows (in p.u.)

Line #	P_{ij}	Q_{ij}	S_{ij}
1-2	- 0.014034	0.0015063	0 014115
1-3	0.410838	0.1008966	0.423046
1-4	0 556199	0.1752962	0.583169
2-3	0.093337	0.0194674	0.095346
2-4	0.466924	0.1403232	0.487553

Spot - Prices ($ / MWh)

Bus #	1	2	3	4
ρ_{pi}	0.0984723	0.0982731	0.0999868	0.10267528
ρ_{qi}	0.0000070	- 0.0000146	0.0003565	0.00081463

Penalty Factors 4500 0 4500.0 4500.0

No of Iterations 7

was evaluated. All lines could be considered in the calculations. However, by selecting only the limiting lines, the computation requirements were significantly reduced. These limiting lines represent the transmission lines that are most readily become overloaded. Since line 1-2 and line 1-3 were the limiting lines, they were used to perform the parametric analysis. Their real power flow ratings are shown in equations (4.2) and (4.3) respectively.

$$P_{12}^{R} = 0\,053623 \text{ pu} \qquad (4\,2)$$

and

$$P_{13}^{R} = 0.461235 \text{ pu} \qquad (4.3)$$

Moreover, P_{12}^{op} and P_{13}^{op} were obtained from Table 4.10 and were given as 0.014033 pu and 0.410838 pu respectively. These values were used in equation (3.27) to calculate $\Delta P_{max,flow}$. The results are shown in equation (4.4).

$$\Delta P_{max,flow} = \begin{bmatrix} 0.039589 \\ 0.050396 \end{bmatrix} \qquad (4.4)$$

The values in equation (4.4) represent the incremental flow constraints Equation (4.5) shows the relationship between the maximum incremental flow constraints and the incremental interchange.

$$[H]\,\Delta P_3 \leq \Delta P_{max,flow} \qquad (4.5)$$

The components of the sensitivity matrix [H] were derived using equation (3.30). They are shown in equation (4.6).

$$[H] = [\frac{\partial P_{flow}}{\partial P_3}] = \begin{bmatrix} \frac{\partial P_{12}}{\partial \theta_1} & \frac{\partial P_{12}}{\partial \theta_2} & \frac{\partial P_{12}}{\partial \theta_3} & \frac{\partial P_{12}}{\partial v_1} & \frac{\partial P_{12}}{\partial v_2} & \frac{\partial P_{12}}{\partial v_3} \\ \frac{\partial P_{13}}{\partial \theta_1} & \frac{\partial P_{13}}{\partial \theta_2} & \frac{\partial P_{13}}{\partial \theta_3} & \frac{\partial P_{13}}{\partial v_1} & \frac{\partial P_{13}}{\partial v_2} & \frac{\partial P_{13}}{\partial v_3} \end{bmatrix} \begin{bmatrix} \frac{\partial \theta_1}{\partial P_3} \\ \frac{\partial \theta_2}{\partial P_3} \\ \frac{\partial \theta_3}{\partial P_3} \\ \frac{\partial V_1}{\partial P_3} \\ \frac{\partial V_2}{\partial P_3} \\ \frac{\partial V_3}{\partial P_3} \end{bmatrix} \quad (4\ 6)$$

The elements of vector $\partial\theta/\partial P_3$ and $\partial V/\partial P_3$ in equation (4.6) were calculated from the third column of the inverse of the Jacobian matrix. The results are shown in equation (4.7).

$$[J_3^{-1}] = \begin{bmatrix} -0.1220 \\ -0.0841 \\ -0.1938 \\ -0\ 0188 \\ -0\ 0107 \\ -0.0354 \end{bmatrix} \quad (4.7)$$

The elements of $\partial P_{flow}/\partial\theta$ and $\partial P_{flow}/\partial V$ in equation (4.6) were calculated using equations (3.33) - (3.36) and the results are given in equations (4.8) and (4.9) respectively.

$$P_{12} = [\,-2.6863 \quad 2\,6863 \quad 0.0 \quad -0.7307 \quad 0.2784 \quad -0.0\,] \tag{4.8}$$

$$P_{13} = [\,-10.493 \quad 0.0 \quad 10.493 \quad -2.410 \quad 0.0 \quad 1.6536\,] \tag{4.9}$$

Using the above calculations, the sensitivity matrix [H] was formed. The values are given in equation (4.10).

$$[H] = \begin{bmatrix} 0.1126 \\ -0.7663 \end{bmatrix} \tag{4.10}$$

Following equations (4.4), (4.5), and (4.10), the sensitivity relationship was established as shown in equation (4.11)

$$\begin{bmatrix} 0.1126 \\ 0\,7663 \end{bmatrix} \Delta P_3 \leq \begin{bmatrix} 0.039589 \\ 0.050396 \end{bmatrix} \tag{4 11}$$

Upon simplifying equation (4.11),

$$\Delta P_3 \leq \begin{bmatrix} 0.3516 \\ 0.0650 \end{bmatrix} \tag{4.12}$$

ΔP_3 describes the distance between the actual line flow and the corresponding line limit. The smaller value indicates that the expected line 1-3 would be overloaded first. Therefore, 0.065 pu was chosen. This value also represents the available amount of energy for sale.

The same procedure was repeated to determine the desired amount of energy to purchase. Following equation (3.40),

$$\begin{bmatrix} 0\ 1126 \\ 0.7663 \end{bmatrix} \Delta P_3 \leq \begin{bmatrix} 0.014033 \\ 0.410838 \end{bmatrix} \tag{4.13}$$

Upon simplifying equation (4.13),

$$\Delta P_3 \leq \begin{bmatrix} 0.12462 \\ 0.53613 \end{bmatrix} \tag{4.14}$$

The smallest value of ΔP_3 was again selected since this indicated that P_{12} would reach zero first Therefore, the amount of energy was 0.125 pu.

4.4.2. Participation Factor Calculation for Generators 1 and 2

The PFC method allows us to estimate how much each corresponding generator must be shifted to serve the new load economically. The PFC is calculated in equation (3.46).

The second order derivatives of both generators were first calculated. They are shown in equations (4.15) and (4.16) respectively.

$$F''(P_{G1}) = 13.4504 \tag{4 15}$$

$$F''(P_{G2}) = 12.1300 \tag{4.16}$$

The total generation to be raised was 6 5 MW (calculated from parametric analysis) and was distributed to generators 1 and 2. The PFC for each unit was calculated using equation (3 46). The results are shown in equation (4.17) and (4.18) respectively.

$$\frac{\partial P_{G1}}{\partial P_{D3}} = 0.47419 \tag{4.17}$$

$$\frac{\partial P_{G2}}{\partial P_{D3}} = 0.5258 \tag{4.18}$$

Equations (4.17) and (4.18) show the fraction of the amount of interchange that each generator should share. These values were multiplied by the amount of interchange ΔP_3 to yield the actual MW value that each unit shared. The results are shown in equations (4.19) and (4.20) respectively.

$$\Delta P_{G1} = 3.0822 \text{ MW} \tag{4.19}$$

and

$$\Delta P_{G2} = 3.4177 \text{ MW} \tag{4.20}$$

ΔP_{G1} and ΔP_{G2} are the estimated amount of energy that generator 1 and 2 should be raised respectively. However, these results must be checked with the limits of the generators If the value of ΔP_{G_1} exceeded the corresponding incremental generator's capability limit, $\Delta P_{G_1}^{max}$, then the amount of interchange was fixed to that limit.

The new operating levels of units 1 and 2 were calculated as shown in equations (4.19) and (4.20) respectively.

$$P_{G1}^{new} = 95.30 + 3.0822 = 98.3822 \text{ MW} \tag{4.19}$$

$$P_{G2}^{new} = 57.43 + 3\ 4177 = 60.8477 \text{ MW} \tag{4.20}$$

These calculations were compared with the actual values obtained from running the OPF at the new load level (P_3^{new} equaled to 0.565 pu). Both results are shown in Table 4.11. The results given in Table 4.11 show a consistency between PFC approach and OPF method. The total difference, $\Sigma(P_{G1} + P_{G2})$, between the two methods yielded 0.1134 MW that was relatively small and could be corrected.

Similarly, the PFC was used to determine the new generation level for the desired amount of purchase, 0.125 pu. The OPF program was rerun at P_{D3} equaled to 0.375 pu. Both calculations are compared in Table 4.12.

Table 4.11. Results using PFC and OPF methods

at P_{D3} equals to 0.565 pu

P_G^{new} (MW)	Participating Factor	OPF Method (Actual Results)	Margin Errors (in MW)
P_{G1}^{new}	98 3822	98 4930	0.110
P_{G2}^{new}	60 8477	60.8504	0.003

Table 4.12 Results using PFC and OPF methods

with P_{D3} equals to 0.375 pu

P_G^{new} (MW)	Participating Factor	OPF Method (Actual Results)	Margin Errors (in MW)
P_{G1}^{new}	89.3729	89.1923	- 0.180
P_{G2}^{new}	50.8553	50.8602	0.005

4.5. Example 5: Implementation of Energy Brokerage Transaction

A 4-utility example was presented to clarify the broker operation. The utilities were interconnected as shown in Figure 4.3. The broker information for each area was obtained from parametric analysis. A 10 MW limit on each tie line was assumed for this example

The conventional brokerage transactions were established following the procedure in section 2.4.3. First, the sale quotation data were arranged in ascending order and the purchase quotation data were arranged in descending order as shown in Table 4.13

The high-low matching procedure began by matching the lowest sell offer with the highest buy bid. Since area 1 and area 3 did not have any bilateral agreements, area 3 was matched with the next lower sell offer. Area 2 was the recipient. The matching proceeded in this way until all offers were processed or all savings were exhausted.

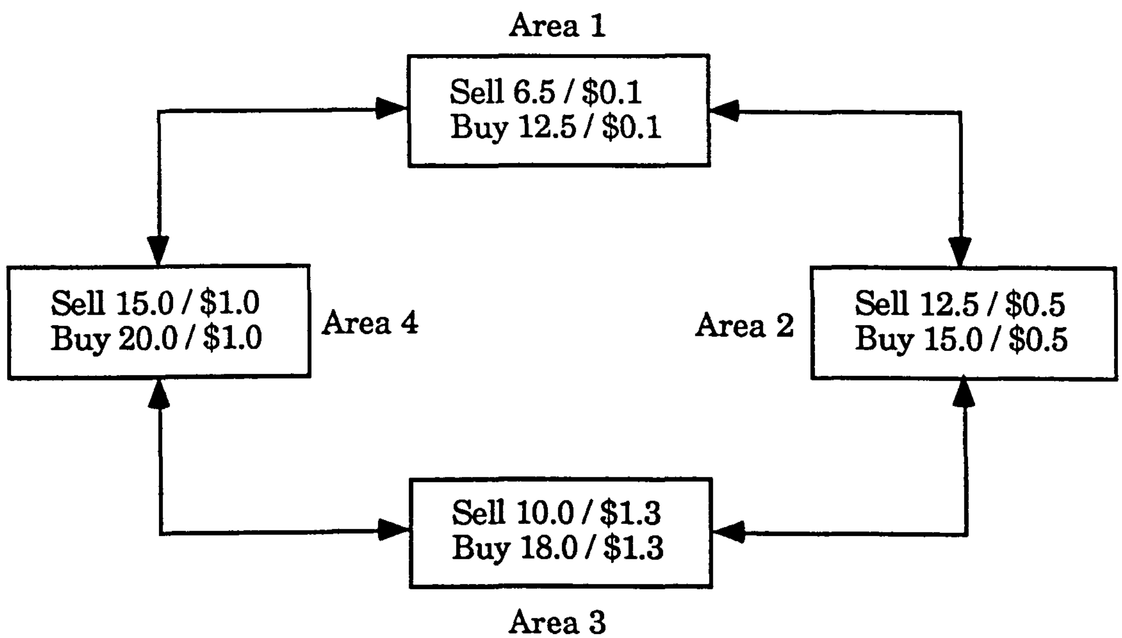

Figure 4.3 A four-area interconnected system.

Table 4.13. Purchase and sale quotes

AREA	Energy (MW)	Sale Quote ($/MWh)	AREA	Energy (MW)	Buy Quote ($/MWh)
1	6.5	0 1	3	18.0	1 3
2	12.0	0.5	4	20.0	1.0
4	15.0	1.0	2	15.0	0.5
3	10.0	1.3	1	12.5	0.1

The conventional broker results were tabulated ın Table 4.14. Note that not all of the available energy could be transferred from area 2 to area 3. The transmission limit had prevented area 2 from doing so. As a result, only 10 MW could be scheduled between the two areas.

Also note that area 4 served as a seller and a buyer simultaneously. Its actual net ınterchange was only 1.5 MW. Moreover, if the net interchange is zero, area 4 will then act as the wheeling party between area 1 and area 3.

Because difficulty arose when brokerage transaction was performed manually because of the amount of data entered, an automated brokerage implementation is more desirable. The amount of savings for each transaction was first calculated. They are shown in Table 4.15. Those transactions with negative values were neglected sınce these indicated that no savings could be realized.

Table 4.14. Conventional brokerage transactions

SELLER	BUYER	Amount (MW)	Sale Quote	Buy Quote	TP ($/MWh)	Savings ($)
2	3	10.0	0.5	1 3	0.90	8.00
4	3	8.0	1.0	1.3	1.15	2 40
1	4	6.5	0.1	1.0	0.55	5 85

Table 4.15. Established transactions

BUYER	SELLER	Buy Quote (λ_b)	Sell Quote (λ_s)	(λ_b - λ_s) $/MWh
3	2	1.3	0.5	0.8
3	4	1.3	1.0	0.3
4	1	1.0	0.1	0.9
4	3	1.0	1 3	- 0.3
2	1	0.5	0.1	0.4
2	3	0.5	1.3	- 0.8
1	2	0.1	0.5	- 0.4
1	4	0.1	1.0	- 0.9

The LP was formulated as shown in equation (4 21). The objective function maximized all the possible per-unit saving calculated in Table 4.15 with respect to the amount of interchange. The LP was solved using GAMS/MINOS and the results are shown in Table 4.16. Note that the conventional brokerage operation and the LP optimization yield the same amount of savings.

MAXIMIZE $F = 0\ 80\ I_{23} + 0.30\ I_{43} + 0.90\ I_{14} + 0.40\ I_{12} + 0.0\ I_{34} + 0\ 0\ I_{32} + 0.0\ I_{21} + 0.0\ I_{41}$

Subject to:

$$
\begin{aligned}
I_{12} + I_{14} &\le 6.50 \\
I_{21} + I_{23} &\le 12.0 \\
I_{32} + I_{34} &\le 10.0 \\
I_{41} + I_{43} &\le 15.0 \\
\\
I_{21} + I_{41} &\le 12.5 \\
I_{12} + I_{32} &\le 15.0 \\
I_{23} + I_{43} &\le 18.0 \\
I_{14} + I_{34} &\le 20.0
\end{aligned}
$$

$$0 \le I_{ij} \le 10.0 \quad i \ne j \qquad i = 1,2,3,4 \qquad j = 1,2,3,4 \tag{4.21}$$

Figure 4.4 shows the direction of the resultant flows of power interchange among the utilities. The net savings for each area were computed and are tabulated in Table 4.17. The total savings that resulted from the broker operation was $ 16.25 for that hour.

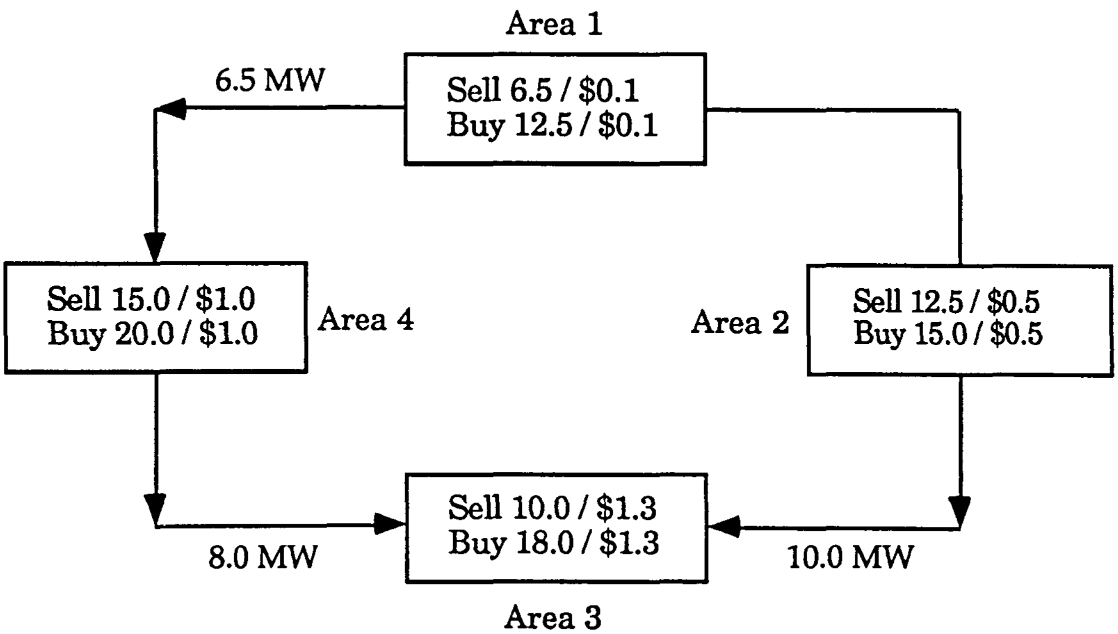

Figure 4.4. Diagram of the resultant flow of power interchange

Table 4.16. Broker results from GAMS/MINOS

SELLER	BUYER	AMOUNT (MW)
2	3	10.0
4	3	8.0
1	4	6.5

Table 4.17. Summary of net savings

AREA	Total Net Savings ($)
1	2 93
2	4.0
3	4.0 + 1.20 = 5 20
4	1.2 + 2.92 = 4.12

4.6. Example 6: Effect of Wheeling Transactions

The same interconnected system from example 4.5 was used to illustrate the wheeling calculation. The broker operation was extended to allow wheeling between non-contiguous utilities. This allowed area 2 to sell its excess energy to area 4 through area 1 or area 3 A certain cost for providing wheeling service was assumed for each area (λ_w). These values are given in Table 4.18. The wheeling cost was first subtracted from the net saving for each pair of transactions and the per-unit saving for each transaction was determined. Those with negative savings were excluded. According to Table 4.18, wheeling should occur between area 1 and area 3 and between area 2 and area 4 respectively.

The LP was formulated according to equation (3.74). This is shown in equation (4.22). The LP problem was solved using GAMS/MINOS and the results are shown in Table 4.19.

The savings that resulted from each transaction are tabulated in Table 4.20 Table 4 21 shows the net saving calculations for each area. These saving calculations assumed that the wheeling cost would be equally divided between the seller and the buyer. The total savings resulted from the wheeling operation was $ 17.24. This demonstrates that additional savings can be achieved through wheeling.

Table 4.18 Calculation of per-unit saving after the wheeling transaction

Buyer	Wheeler	Seller	λ_b	λ_s	λ_w	λ_b-λ_s-λ_w
3	2	1	1.3	0.1	0.010	1.190
3	4	1	1.3	0.1	0.020	1.180
4	1	2	1 0	0 5	0.005	0.495
4	3	2	1 0	0.5	0.015	0.485
2	1	4	0.5	1.0	0.005	- 0.505
2	3	4	0.5	1.0	0.015	- 0.515
1	2	3	0.1	1 3	0.010	- 1.210
1	4	3	0.1	1.3	0.020	- 1.220

MAXIMIZE $\quad F = 0.80\, I_{23} + 0.30\, I_{43} + 0\,90\, I_{14} + 0.40\, I_{12}$

$$+\ 0.0\, I_{34} + 0.0\, I_{32} + 0\,0\, I_{21} + 0\,0\, I_{41}$$

$$+\ 1.19\, I_{123} + 1.18\, I_{143} + 0.495\, I_{214} + 0\,485\, I_{234}$$

Subject to:

$$I_{12} + I_{14} + I_{123} + I_{143} \leq 6.50$$

$$I_{21} + I_{23} + I_{214} + I_{234} \leq 12.0$$

$$I_{32} + I_{34} + I_{321} + I_{341} \leq 10.0$$

$$I_{41} + I_{43} + I_{412} + I_{432} \leq 15\,0$$

$$I_{21} + I_{41} + I_{321} + I_{341} \leq 12.5$$

$$I_{12} + I_{32} + I_{412} + I_{432} \leq 15.0$$

$$I_{23} + I_{43} + I_{123} + I_{143} \leq 18.0$$

$$I_{14} + I_{34} + I_{214} + I_{234} \leq 20.0$$

$$0 \leq I_{ij} + I_{ik} \leq 10\,0$$

$$0 \leq I_{ij} + I_{kj} \leq 10.0$$

$$i \neq k \neq j \qquad i = 1,2,3,4 \qquad k = 1,2,3,4 \qquad j = 1,2,3,4$$

(4.22)

Table 4.19. Wheeling results from GAMS/MINOS

Seller	Wheeler	Buyer	Energy (MW)
1	-	4	6 5
2	-	3	10.0
4	-	3	8.0
2	1	4	2.0

Table 4.20. Savings from wheeling operation

SELLER	BUYER	Amount (MW)	Sale Quote	Buy Quote	Wheel Quote	$(\lambda_b-\lambda_s) - \lambda_w{}^d$ ($/MWh)	Savings ($)
2	3	10.0	0.5	1.3	0 90	0 800	8 00
2	4	2.0	0 5	1.3	0 9	0.495	0.99
4	3	8.0	1.0	1.3	1.15	0.300	2.40
1	4	6 5	0.1	1.0	0 55	0 900	5 85

[d] Wheeling charge is first subtracted if any wheeling agreements exist.

Table 4.21. Summary of net savings

AREA	Total Net Savings ($)
1	2.93
2	4.0 + 0.50 = 4.50
3	4 0 + 1.20 = 5.20
4	1.2 + 2.92 + 0.49 = 4.61

CHAPTER V. CONCLUSION

5.1. Summary

This research provides the ground work for evaluating economy power interchange. The necessary indices for performing the economic transaction were obtained from an OPF solutions. The OPF aids in performing a complete analysis of the power system. It alleviates real and reactive power flow overloads, eliminates overvoltage problems, and continually balances generation and load in the most economical way. The OPF uses a new class of optimization called the augmented Lagrangians. The good convergence properties enable this algorithm to handle complex non-linear constraints. One advantage of this method is that it provides a "close-to-optimum" solution whenever some constraints cannot be tolerated. Another attractive feature is that the spot prices are automatically calculated once the OPF problem is solved. In addition, the AL problem is easy to formulate. It does not require piecewise linearization as in the LP formulation.

One major drawback of the AL approach is the need to reformulate and resolve the power system equations when changes are made to the components of the power system. Since the convergence depends highly upon the efficiency of the unconstrained AL minimization, a high performance technique for solving the unconstrained minimization should be considered when attempting to solve a large-scale power system.

The amount of interchange is determined by performing a parametric analysis. This method uses the linearized sensitivity relationship of the power system to determine the maximum incremental export/import capabilities at the interchange buses with respect to the transmission constraints. The amounts of export and import energy represent the energy for sale and purchase respectively. They are determined and distributed to all generator buses in the system using a "participating factor", thus allowing the system to continue to operate economically. This is a major accomplishment over the classical approach since additional power export/import is no longer absorbed only by the slack generator.

The economy power interchange was performed using the interchange brokerage operation. The broker scheme was formulated as a linear programming model and was solved using GAMS/MINOS. Savings were computed and distributed equally based on the split-the-saving formula.

5.2. Recommendations for Future Work

The probability concept plays an important role in every aspect of power flow studies Since the power system conditions at a given time in the future cannot be specified precisely, we should have more confidence in the solution provided by a probabilistic OPF. The research should continue to improve the AL OPF technique by including probabilistic considerations of potential outages. This will allow us to analyze not only the complex power system under normal operating conditions, but also the effects of power system faults. Then, the OPF parameters would be represented as random variables.

A brokerage operation has been successfully modeled in performing the economy power interchange. The potential savings and non-monetary benefits might further increase if the broker scheme were extended to include long-term transactions. This type of broker transaction is called superbroker and requires a unit commitment evaluation. The AL algorithm is a promising tool to do this. Recently, Electricité de France (EDF) has reported that its new software that implements AL algorithm works very well in solving the daily generation scheduling problem [18]. Future work might apply this robust algorithm to solve the superbroker scheme.

BIBLIOGRAPHY

[1] Wood, Allen J. and Wollenberg, Bruce F., "Power Generation, Operation, and Control," John Wiley & Sons, Inc., New York, 1984

[2] Landgren, G.L., Terhune, H.L., and Angel, R.K., "Transmission Interchange Capability - Analysis by Computer," IEEE PAS 1971, pp. 2405-2414.

[3] Landgren, G.L. and Anderson, S.W., "Simultaneous Power Interchange Capability Analysis," IEEE PAS 1972, pp. 1973-1986.

[4] "Simultaneous Transfer Capability: Direction for Software Development," EPRI EL-7351 Final Report, August 1991.

[5] Stott, B., Alsac, O., and Monticelli, A.J., "Security Analysis and Optimization," Proceedings of the IEEE, Vol-75, No. 12, December 1987, pp. 1623-1644.

[6] Alsac, O., Bright, J., Prais, M., and Stott, B., "Further Developments in LP-Based Optimal Power Flow," IEEE Trans. on Power Systems, Vol-5, No. 3, August 1990, pp. 697-711.

[7] Papalexopoulos, A.D., Imparato, C.F., and Wu, F.F., "Large Scale Optimal Power Flow: Effects of Initialization, Decoupling and Discretization," IEEE PWRS 1989, Vol-4, No. 2, pp. 748-759.

[8] Miller, J.M., Balmat, B.M., Morris K.N. et al, "Operating Problems with Parallel Flows," committee report submitted to IEEE PWRS 226-1, 1991.

[9] Mescua, J., "A Decoupled Method for Systematic Adjustments of Phase-Shifting and Tap-Changing Transformers," IEEE PAS, Vol-104, No 9, September 1985, pp. 2315-2321.

[10] Schweppe, Fred C., Caramanis, M. C., et. al., "Spot Pricing of Electricity," Kluwer Academic Publishers, Boston 1988.

[11] Doty, K. W., McEntire, P. L., "An Analysis of Electric Power Brokerage System," IEEE PAS 1982, Vol-101, No. 2, pp. 389-396.

[12] Pierre, Donald A. and Lowe, Michael J., "Mathematical Programming Via Augmented Lagrangians," Addison-Wesley Publishing Company, Boston, 1975.

[13] Wismer, David A. and Chattergy, R., "Introduction to Nonlinear Optimization," North-Holland Publishing Company, Amsterdam 1978.

[14] Santos, A., Deckman, S., and Soares, S., "A Dual Augmented Lagrangian Approach for Optimal Power Flow," IEEE Trans. on Power Systems, Vol-3, No 3, August 1988, pp. 1020-1025.

[15] Mamandur, K.R.C. and Berg, G.J., "Economic Shift in Electric Power Generations with Line Flow Constraints," IEEE PAS, Vol-97, No. 5, Sept/Oct 1978, pp. 1618-1625.

[16] Fahd, George, Richards, Daniel A., and Sheblé, G. B., " The Implementation of an Energy Brokerage System using Linear Programming," IEEE Trans. on Power System, Vol-7, No. 1, February 1992, pp. 90-96

[17] Fahd, G. and Sheblé, G B., "Optimal Power Flow Emulation of Interchange Brokerage System using Linear Programming," Paper SM515-7 to be presented at the IEEE/PES Summer Meeting 1991.

[18] Batut, J. and Renaud, A., "Daily Generation Scheduling Optimization with Transmission Constraints: A New Class of Algorithms," Electricité de France, Clamart, France 1991.

APPENDIX A. GENERATOR OPERATING LIMITS

Practically speaking, a generator-operating limits are constrained by its rotor heating and stator end-core heating limits. Operating the generator beyond these limits may cause severe damage to the windings. The information on the rotor-heating limit and stator end-core stability limit is necessary to obtain an accurate representation of the operating limits and are usually provided by the manufacturer. A crude way to calculate the reactive power operating limits is as follows:

1. First, interpret the relationship of Q_g versus P_g **geometrically**.

   The rotor heating and stator end–core heating limit and the real power operating limit conditions are plotted on a Q_g vs P_g plane.

2. Using the given value P_g^{min}, draw a vertical line across P_g^{min} until it hits the rotor and stator end-core heating limits to obtain two points, (a, P_g^{min}) and $(-b, P_g^{min})$. Apply the same procedure using P_g^{max} to obtain the other two points, (c, P_g^{max}) and $(-d, P_g^{max})$

3. Use equation (a.1) to find the line that passes through two points.

$$\frac{y - y_0}{y_1 - y_0} = \frac{x - x_0}{x_1 - x_0} \qquad \text{(a.1)}$$

By inserting (a,P_g^{min}) and (c,P_g^{max}) into equation (a.1), we obtain

$$Q_g^{max} = r + s\,P_g^{op} \qquad \text{(a.2)}$$

Similarly, inserting $(-b,P_g^{min})$ and $(-d,P_g^{max})$ into equation (a.1) yields

$$Q_g^{min} = u + w\,P_g^{op} \qquad \text{(a.3)}$$

Figure A.1 depicts the geometrical figure of the generator operating limits.

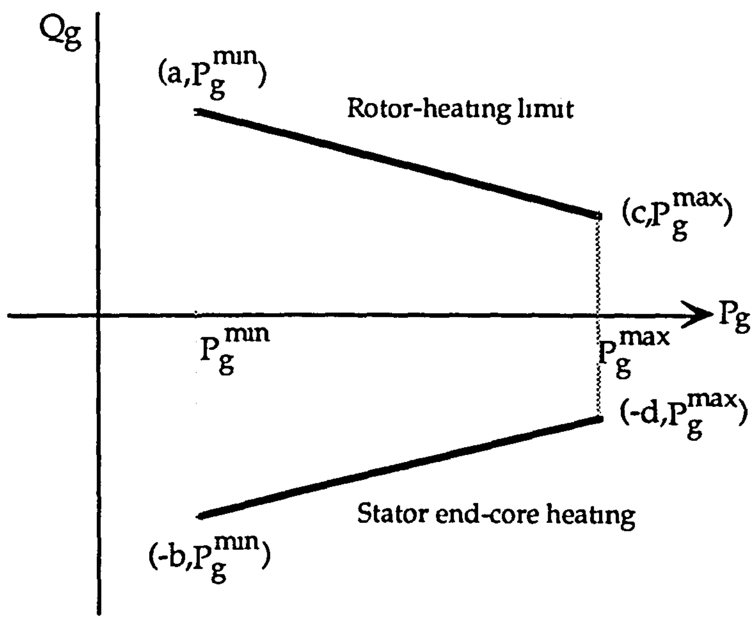

Figure A.1 Generator-operating limits

APPENDIX B. FORMULATION OF JACOBIAN SUBMATRIX

Define the linearized power flow injection relationships as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial v} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial v} \end{bmatrix}_{\theta, V} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}$$

Let

$$T_{ij} = G_{ij} \operatorname{Cos}(\theta_i - \theta_j) + B_{ij} \operatorname{Sin}(\theta_i - \theta_j)$$

$$U_{ij} = G_{ij} \operatorname{Sin}(\theta_i - \theta_j) - B_{ij} \operatorname{Cos}(\theta_i - \theta_j)$$

$$G_{ii} = \sum_{j \varepsilon t(i)} (G_{ij} + G_{sij})$$

$$B_{ii} = \sum_{j \varepsilon t(i)} (B_{ij} + B_{sij})$$

t(i) denotes the set of buses connected to bus i

Then each submatrix of the Jacobian matrix can be expressed as:

$$\frac{\partial P_i}{\partial \theta_i} = V_i \sum_{\substack{k \varepsilon t(i) \\ k \neq i}} V_k U_{ik}$$

$$\frac{\partial P_i}{\partial \theta_j} = - V_i V_j U_{ij}$$

$$\frac{\partial P_i}{\partial V_i} = 2\, V_i\, G_{ii} \; - \sum_{\substack{k \varepsilon t(i) \\ k \neq i}} V_k\, T_{ik}$$

$$\frac{\partial P_i}{\partial V_j} = -\, V_i\, T_{ij}$$

$$\frac{\partial Q_i}{\partial \theta_i} = -\, V_i \sum_{\substack{k \varepsilon t(i) \\ k \neq i}} V_k\, T_{ik}$$

$$\frac{\partial Q_i}{\partial \theta_j} = V_i\, V_j\, T_{ij}$$

$$\frac{\partial Q_i}{\partial V_i} = -\, 2\, V_i\, B_{ii} \; - \sum_{\substack{k \varepsilon t(i) \\ k \neq i}} V_k\, U_{ik}$$

$$\frac{\partial Q_i}{\partial V_j} = -\, V_i\;\, U_{ij}$$